PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 73 MAY, 1947 No. 5

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum Price $1.00 per copy

Copyright, 1947, by the AMERICAN SOCIETY OF CIVIL ENGINEERS

Printed in the United States of America

	Published	Discussion closes
Peterson, F. G. Eric. Effect of Stress Distribution on Yield Points Discussion in Sept., Nov., Dec., 1946, Mar., 1947	Apr., 1946	Closed*
Baker, Donald M. Some Thoughts on Engineering Education Discussion in Sept., Oct., Dec., 1946, Feb., May, 1947	Apr., 1946	Closed*
Rose, Edwin. Thrust Exerted by Expanding Ice Sheet Discussion in Dec., 1946	May, 1946	Closed*
Dell, George H. Moment-Stiffness Relations in Continuous Frames with Prismatic Members Discussion in Oct., 1946, Apr., 1947	May, 1946	Closed*
Bermel, Karl J., and Sanks, Robert L. Model Study of Brown Canyon Debris Barrier Discussion in Nov., Dec., 1946	May, 1946	Closed*
Jacob, C. E. Effective Radius of Drawdown Test to Determine Artesian Well Discussion in Nov., Dec., 1946, Feb., 1947	May, 1946	Closed*
Moore, R. L. Observations on the Behavior of Aluminum Alloy Test Girders	June, 1946	Closed*
Hansen, Howard J. Design of Plywood I-Beams Discussion in Nov., 1946, Jan., May, 1947	June, 1946	Closed*
Tommerup, Carl C. H. Rigid-Frame Structures Subject to Nonuniform Thermal Action Discussion in Nov., 1946, Jan., Mar., 1947	June, 1946	Closed*
Middlebrooks, T. A., and Jervis, William H. Relief Wells for Dams and Levees Discussion in Oct., Dec., 1946, Jan., Feb., May, 1947	June, 1946	Closed*
Stevens, J. W. Truck Speed and Time Loss on Grades Discussion in Dec., 1946, Mar., 1947	Sept., 1946	Closed*
Underwood, P. H. Space Resection Problems in Photogrammetry Discussion in Dec., 1946, Jan., May, 1947	Sept., 1946	Closed*
McGuinness, Charles L. Recharge and Depletion of Ground-Water Supplies Discussion in Mar., Apr., 1947	Sept., 1946	Closed*
de Vries, Karl. Strength of Beams as Determined by Lateral Buckling Discussion in Dec., 1946, Feb., Mar., Apr., May, 1947	Sept., 1946	Closed*
Ward, C. N., and Hunt, Henry J. Correction of Tailwater Erosion at Prairie Du Sac Dam Discussion in May, 1947	Oct., 1946	Closed*
Benscoter, Stanley U. Matrix Analysis of Continuous Beams Discussion in Feb., Apr., 1947	Oct., 1946	Closed*
Machis, Alfred. Experimental Observations on Grouting Sands and Gravels Discussion in Feb., Mar., 1947	Nov., 1946	July 1, 1947
Symposium: Cleaning and Grouting of Limestone Foundations, Tennessee Valley Authority Discussion in Mar., May, 1947	Dec., 1946	July 1, 1947
Matthes, Gerard A. Mississippi River Cutoffs Discussion in Mar., 1947	Jan., 1947	July 1, 1947
Tan, Ek-Khoo. Stability of Soil Slopes Discussion in May, 1947	Jan., 1947	July 1, 1947
Muldrow, W. C. Forecasting Productivity of Irrigable Lands Discussion in May, 1947	Feb., 1947	July 1, 1947
Freudenthal, Alfred M. Reflections on Standard Specifications for Structural Design	Feb., 1947	July 1, 1947
Blaisdell, Fred W. Development and Hydraulic Design, Saint Anthony Falls Stilling Basin	Feb., 1947	July 1, 1947
DuVal, Miles P. The Marine Operating Problems, Panama Canal, and the Solution Discussion in Apr., 1947	Feb., 1947	Uncertain
Claybourn, J. G. Sea Level Plan for Panama Canal Discussion in Apr., 1947	Feb., 1947	Uncertain
Hrennikoff, Alexander. Theory of Inelastic Bending with Reference to Moment Design	Mar., 1947	Aug. 1, 1947
Hsu Shih-Chang. Beam Deflections by Second and Third Moments	Mar., 1947	Aug. 1, 1947
Ryder, Frederick L. A Rational Explanation of Column Behavior Discussion in May, 1947	Mar., 1947	Aug. 1, 1947
Report. Organization, Financing, and Administration of Sanitary Districts: Final Report of the Committee of the Sanitary Engineering Division	Mar., 1947	
Rathbun, J. Charles, and Cunningham, C. W. Continuous Frame Analysis by Elastic Support Action	Apr., 1947	Sept. 1, 1947
Hickox, G. H., Peterka, A. J., and Elder, R. A. Friction Coefficients in a Large Tunnel	Apr., 1947	Sept. 1, 1947
Sturm, R. G. Stability of Thin Cylindrical Shells in Torsion	Apr., 1947	Sept. 1, 1947
Report: Water Supply Engineering: Report of the Committee of the Sanitary Engineering Division for the Two Years Ending December 31, 1946	Apr., 1947	

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR MAY, 1947

PAPERS

PAGE

Tests of Timber Structures from Golden Gate International Exposition.
By a Committee of the San Francisco (Calif.) Section, ASCE, on Timber Test Program 571

New Formulas for Stresses in Concrete Pavements of Airfields.
By H. M. Westergaard 687

DISCUSSIONS

Design of Plywood I-Beams.
By Howard J. Hansen 703

Some Thoughts on Engineering Education.
By Henry B. Lynch, and Alfred R. Golzé 705

Correction of Tailwater Erosion at Prairie du Sac Dam.
By George P. Steinmetz, Louis M. Laushey, Abraham Streiff, and R. H. Keays .. 709

Space Resection Problems in Photogrammetry.
By George H. Dell 718

Cleaning and Grouting of Limestone Foundations, Tennessee Valley Authority: A Symposium.
By Robert A. Laurence 721

Stability of Soil Slopes.
By Paul Baumann, and Gregory P. Tschebotarioff 723

Forecasting Productivity of Irrigable Lands.
By Arthur F. Johnson, H. W. Van Loo, and Will H. Noble 729

Strength of Beams as Determined by Lateral Buckling.
By H. G. Brameld 733

CONTENTS FOR MAY, 1947 (*Continued*)

PAGE

Relief Wells for Dams and Levees.
By Reginald A. Barron, Preston T. Bennett, Charles I. Mansur, and S. J. Johnson 735

A Rational Explanation of Column Behavior.
By R. Robinson Rowe, and George Winter 756

A list of "Current Papers and Discussions" may be found on the page preceding the table of contents

The Society is not responsible for any statement made or opinion expressed in its publications

HINTS TO AUTHORS

Those who are planning papers for submission to PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Manager, Technical Publications, 33 W. 39th St., New York 18, N. Y.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

TESTS OF TIMBER STRUCTURES FROM GOLDEN GATE INTERNATIONAL EXPOSITION

BY A COMMITTEE OF THE SAN FRANCISCO (CALIF.) SECTION, ASCE, ON TIMBER TEST PROGRAM

COMMITTEE

HAROLD B. HAMMILL, *Chairman*

HENRY J. DEGENKOLB
HENRY D. DEWELL (deceased March 20, 1946)
JOHN J. GOULD
LEON H. NISHKIAN

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 1, 1947.

CONTENTS

	PAGE
Synopsis	573
A. Foreword	574
B. History	575
C. General Features	578
I. Operations and Computations	578
II. Safety Factor	581
III. Nomenclature	582
D. Detailed Report of Tests	585
I. Truss Tests	585
II. Joint Tests	635
III. Column Tests	669
IV. Plywood Tests	671

Synopsis

Seventeen trusses, thirty-seven joints of structural frames, and three 15-ft struts, all of which had served for some 2½ years as structural parts of the Golden Gate International Exposition buildings at San Francisco, Calif., were removed from these buildings when they were dismantled, stored for some weeks in the open air, and later were tested for failure in a large testing frame constructed especially for this purpose. Also, one hundred and three laboratory specimens, constructed to find the lateral resistance of nails in plywood and the shearing and buckling strength of plywood panels, were tested.

The trusses were of various types, spans, and sizes. Thirteen were entirely of timber. Seven of these had connections framed with split rings or shear plates, and the other six had connections of volts and washers. Four of the seventeen trusses were of the rod and block type, with compression diagonals of wood and with tension members of steel rods. The thirty-seven joints with shear plates, split rings, and bolts were of various types: Seventeen with members connected by steel gusset plates and shear plates, one with plywood gusset plates and bolts, two joints and four splices with split rings, six bolted spliced joints of girders resisting moment and shear, two butt-block end joints of trusses, three joints with steel straps and pins, two bolted joints with steel straps, and four laboratory-built tension splices or compression splices with plywood splice plates and split rings. One spaced column, two T-section columns and struts, eighty-two nail fastenings in plywood, three plywood panels in shear, and eighteen plywood panels subjected to buckling were also tested.

Loads were applied with calibrated hydraulic jacks. Deflections of each truss as a whole, as well as deformations of its component parts, were measured during the progressive loadings. Likewise, deflections and deformations were measured in the various joints, columns, and plywood tests. Numerous photographs illustrate the features of interest in the tests.

Loads and stresses were computed after each test. For example, the strength or weakness of each truss and its parts was investigated theoretically to correlate test phenomena and structural theory.

A. FOREWORD

This report presents: Tests and studies of the load-carrying capacity, strength, deflection, and deformation of members of full-sized wood trusses; similar tests of joints of wood trusses and structural frames, splices of wood girders, and built-up wood columns; tests of the buckling and shearing strength of plywood panels; and tests of the lateral strength of nail fastenings in plywood. All the trusses had supported the roofs or floors of the Golden Gate International Exposition buildings for 2 or more years, and were made available for testing when the buildings were dismantled in 1941. All test specimens were taken intact from these buildings, or were fabricated of material from them. Most of the trusses were framed with modern timber connectors (that is, split rings and shear plates) although some had bolted joints without connectors and others were framed with web members of steel rods and wood struts.

These tests and their analyses are believed to be unique in presenting a comprehensive study of actual wood trusses and structural members, not designed and constructed as test specimens. The structural members had functioned as parts of buildings for 2½ years; they were generally of good material; they had been designed by competent structural engineers; they had been built under ordinary field circumstances and under varying conditions of inspection; and, finally, they were tested as taken from the buildings with such defects as normally exist, without tightening loose bolts and (except in a few cases specifically noted, where trusses were damaged while being taken down) without correcting any deficiencies resulting from service.

The original purpose was to take advantage of an opportunity seldom available; namely—(1) to test entire trusses in place in the buildings, under normal conditions of use as nearly as possible with the various defects of workmanship accompanying commercial practice; (2) to study the relation between laboratory tests of individual members or joints and their combinations as found in framed structures; (3) to determine whether factors had been overlooked in design, construction, inspection, or maintenance on which more information was needed; (4) to note the effect of shrinkage and other factors that develop in service; and (5) to test individual joints with multiple connectors and with several members forming concentric or eccentric connections.

It became necessary, however, to demolish many of the buildings before it was possible to test their trusses in place. As a substitute, although not so desirable, the individual trusses were tested as isolated specimens in a specially constructed testing frame. Some of these trusses were of types in which the influence of tension perpendicular to the grain was an important factor. These tests revealed for the first time the importance of considering this stress combined with horizontal shear or other stress in eccentrically connected joints.

Neither the assumed loads nor the unit stresses for which the trusses were designed are known to the committee. The trusses and the structures all functioned satisfactorily. As a result, it is entirely possible that, in the case of some of the critical conditions found in testing these joints and trusses to ultimate load, the designing engineer may have refrained from utilizing the full

strength of a member or joint, fully realizing that eccentricities (for example) produced additional stress which required reduction of stress from direct axial loading.

Phenomena revealed by the tests and subsequent analysis and discussions of these phenomena may be noted as follows:

1. The inadequacy of small individual tests as a premise for determining the behavior and strength of full-sized structures;
2. The importance of considering the effect of combined stresses and the need for further tests and more data on this subject;
3. The importance of adequate supervision or inspection to assure accuracy of fabrication and erection;
4. The tendency of a truss chord to rotate when tension members are connected on one side and compression members on the other, resulting in bending or bowing of web members flatwise;
5. The need of keeping adjacent faces of members in contact by tightening nuts on bolts when it has been necessary to use lumber that has not been seasoned to the moisture content to which it will come during service;
6. The desirability of reducing end checking and splitting;
7. The beneficial effect of cross bolts or stitch bolts, or threaded spikes or dowels, or some kind of adjustable strapping in offsetting the effect of splitting;
8. The importance, where shear plates are used, of avoiding oversized bolt holes, either by error or by reaming to permit passage of bolts through holes bored improperly in the wood;
9. The importance of straight end cuts, without reentrant angles; and
10. The importance of ample end distances in tension members for ring connectors and shear plates.

The main objective of the tests was to determine a factor of safety for the structures tested, for comparison with factors of safety for individual test specimens. This objective was not realized fully, and any comparison of the ratios of ultimate loads as related to assumed safe working loads is misleading unless examined in the light of the material presented in Section C–II under "Safety Factor" and with due attention to individual test comments.

Of the twenty tests made, on sixteen trusses, the results show that: Five failed because of rupture of the wood members at joint details; six failed at bolted joints; and nine failed at connector joints, because of types or combinations of stresses not anticipated.

B. HISTORY

In October, 1940, preliminary work was started on the Timber Test Program at Treasure Island in San Francisco. The program was conceived by John J. Gould, M. ASCE, who had been chief structural engineer of the Exposition, and it was adopted by the San Francisco Section of the Society through the interest and recommendation of the late Frederick H. Fowler (Past-President, ASCE), Harold B. Hammill (M. ASCE), and Walter Dreyer (M. ASCE)—all Past-Presidents of the San Francisco Section. The sponsorship of the University of California (at Berkeley) was obtained through the cooperation

of C. T. Wiskocil, M. ASCE, and W. C. Pomeroy, registrar of the university, then acting as administrative officer of the Works Progress Administration (WPA) for the university. The San Francisco Section appointed a Timber Testing Committee with L. H. Nishkian, M. ASCE, as chairman. When he resigned the chairmanship because of press of private work and extended absence from San Francisco, Mr. Hammill succeeded him. The other members of the committee were Mr. Gould and the late Henry D. Dewell, M. ASCE, and Henry J. Degenkolb, Assoc. M. ASCE. Mr. Gould was retained as technical director of the program by the committee, which functioned in cooperation with the various timber industries who furnished the funds required for the work. Mr. Degenkolb was appointed assistant technical director. Byron L. Nishkian, Jun. ASCE, assisted Messrs. Gould and Degenkolb in the first few months of the program. Mr. Gould, in a part-time capacity and Mr. Degenkolb in a full-time capacity, directed all the technical and administrative phases of the program and prepared the preliminary report, which was edited by Mr. Dewell and then reviewed by the entire committee.

Industries that contributed to the program include the West Coast Lumbermen's Association; National Lumber Manufacturers' Association; Douglas Fir Plywood Association; Timber Engineering Company, Inc., of Washington, D.C.; Timber Engineering Company of California; Timber Structures of Portland, Ore.; Summerbell Roof Structures of Los Angeles, Calif.; the Western Pine Manufacturing Company of Spokane, Wash.; Henry Mill and Timber Company of Tacoma, Wash.; and the Northwest Bolt and Nut Company of Seattle, Wash. The following were instrumental in the organization of the project and kept in close touch with all progress: The late Chester J. Hogue, M. ASCE, in charge of technical service, West Coast Lumbermen's Association; A. C. Horner, Assoc. M. ASCE, consulting engineer, Timber Engineering Company of California; and the late J. E. Mackie, Assoc. M. ASCE, then western manager, National Lumber Manufacturers' Association.

The San Francisco Section and the committee owe these three gentlemen particular gratitude, for they more than once were instrumental in securing additional funds when such financial assistance was sorely needed. Finally, Messrs. Mackie and Horner were generous in supplying the services of their office staffs in the preparation of this report. The Division of Timber Mechanics of the Forest Products Laboratory in Madison, Wis., was very helpful in giving consultation, in answering inquiries, and in reviewing this report.

The late John A. Newlin, M. ASCE, formerly chief of the Division of Timber Mechanics of the Forest Products Laboratory, made a special trip to San Francisco for the purpose of witnessing the tests and advising with the committee. Because his death intervened, the report has not had the benefit of his criticism, which would have been of great value because of Mr. Newlin's knowledge of timber mechanics and wide experience in timber testing. Fortunately, notes made by him have been materially helpful.

Preliminary work began in October, 1941, and consisted principally of the selection of specimens, investigation of methods of testing, application for WPA assistance, and general review of the condition of the Exposition buildings for availability of test specimens. Many studies and drawings were made

by the technical director and his assistant to determine the most advantageous and economical tests and procedures. Unfortunately, various setbacks beyond the control of the committee hindered progress. Some of the buildings were demolished by the wrecking contractors for the Exposition before any arrangements could be made for securing the required specimens. This circumstance was partly offset by the cooperation of the contractors, who donated or allowed various specimens to be taken in return for payment of labor and equipment costs for the extra trouble involved in setting these specimens aside.

The proposal of the United States Navy to take over all of Treasure Island and its buildings caused several revisions in the list of specimens considered necessary for a satisfactory fulfilment of the program. Later, the Navy did take over the island, necessitating the dismantling of the laboratory built for the test program and removal of its equipment to a new location in San Francisco. Finally, the WPA assistance was secured later than had been anticipated. However, to counteract several of those difficulties, the program was fortunate in having the splendid cooperation of Clarence E. Seage, Assoc. M. ASCE, who was chief engineer of the California Commission for the Exposition and who was instrumental in securing many specimens. Ralph G. Wadsworth, M. ASCE, then deputy WPA administrator for Northern California, was especially helpful in the organization of the project and in expediting the work on the many necessary preliminary details.

On February 17, 1941, the Timber Test Program, known as a WPA project, was opened officially. For several months the major part of the project was the collection of specimens. It had been hoped that these could be assembled simultaneously with the testing. However, the rapidity of the demolition made it necessary to obtain the specimens during this work and to delay the testing. Simultaneously, material was furnished to two senior students of the University of California for a special study in timber supervised by the technical director and his assistant. In May, 1941, tests were made on the Cavalcade cantilever structure; and, in June, a building of 2,800 sq ft was constructed, with a 500,000-lb testing frame, all as described in this report.

During the last few days of June, 1941, the first of the trusses was tested—the San Francisco Building truss; and, shortly thereafter, in turn, the Chinese Building trusses, Alameda Contra-Costa Counties Building trusses, Cavalcade truss, French Building truss, Merry-Go-Round trusses, and two rigid-frame legs from the Brazil Building. At this time, because of the national emergency of World War II, it became necessary to move all timber testing equipment and specimens from Treasure Island to San Francisco. In this emergency, entailing unexpected expenses, a building was loaned to the committee by the City of San Francisco, through the cooperation of the late A. D. Wilder, M. ASCE, then director of public works, and James H. Turner, M. ASCE, then civil engineer of the Public Utilities Commission and later manager and chief engineer of the Hetch Hetchy Water Supply, Power and Utilities Engineering Bureau. There was not enough room in this building, however, for all the specimens and equipment, but through the courtesy of J. F. Marias and T. Twohig, then president and rental manager, respectively, of the State Board of Harbor Commissioners, space was obtained at the Ferry Building for additional storage.

Considerable effort and time were expended in making arrangements for moving, and in settling the new building, after which the remaining tests were conducted. Simultaneously, the office staff began and completed the compilation of the data on which this report is based.

Much credit is due Messrs. Degenkolb and Byron Nishkian for the excellent and full photographic record of the tests, only a part of which appears in this report.

C. GENERAL FEATURES

I. Operations and Computations

Equipment.—The necessity of strict economy affected not only the equipment but also the methods of testing. Although the supply of WPA labor was sufficient, cash reserves were at a minimum. The equipment as planned had to be capable of testing 70-ft trusses, individual joints of other structural frames, plywood panels, and 30-ft columns. In the tests of trusses, the loads had to be applied at various points to simulate actual loading conditions. To meet these varied requirements, a large testing structure, or testing frame of 500,000-lb capacity—one that could be used partly or entirely for all the required operations—was designed by the technical staff and then built with WPA labor, using timbers from the dismantled Exposition buildings, connected with 4-in. split rings and ¾-in. bolts.

Hydraulic jacks were used for applying, measuring, and controlling the loads. A pump, flexible hose, two 20-ton jacks, and a 50-ton jack were purchased, along with a large gage for measuring the oil pressure in the jacks. Two 200-ton jacks were loaned to the project by D. J. and T. Sullivan, contractors for moving structures, for testing the heavy specimens. All this equipment was checked and calibrated at the testing laboratory of Abbot A. Hanks, Inc., of San Francisco. Calibration curves were plotted, and studies were made on various factors that might affect their accuracy. Among these factors were the effect of eccentric loading of the plungers, the direction of movement of the plunger, and the effect of using different hose lengths between the pump and the rams.

In loading specimens, care was exercised in centering jacks and loads. As a result, it was felt that the applied loads were measured well within 3% of their true values. A slight source of possible error was that the pressure gage read only to divisions of 25 lb per sq in. for a range of 10,000 lb per sq in. of oil pressure in the jacks. In the truss tests, two or three jacks were used simultaneously with one pump and gage to provide several loading points on a truss, and dividing beams spread the load over several panel points as desired. For this purpose, special Y-connections provided several sources of oil pressure for the jacks.

Twenty gages with Ames dials or Starrett dials, reading directly to 0.001 in., with a range of 0.3 in., were used. In the tests on the trusses, it was desired to read many more deformation points than there were dial gages. A special gage was developed for this purpose consisting of two small metal angles, one fastened to each side of the deformation point. Small ball points were attached on the outside of the outstanding legs of the angles, and the distance

between points was then measured with a micrometer. By using the rachet of the micrometer, a uniform pressure was placed on the ball points at each reading so that any deformations due to bending of the angle legs would be constant. These gages had the advantage that, with the micrometers used, they had a range of about 1 in., reading to 0.001 in. By using vernier calipers, this range could be extended to about 5 in., reading to 1/128 in. and to any desired larger range by measuring with a ruler fitted with the heads of two carpenter's squares. The principal disadvantage of this special gage was the time consumed in making the required number of readings. To offset this disadvantage, in several places where an accuracy of 0.01 in. was considered sufficient, simpler gages were made, using a multiplying arm with a large scale. These measuring devices were calibrated with the dial gages and had a range of about $2\frac{1}{2}$ in.

Testing Procedure.—In most of the tests, the actual readings were made by WPA workers, none of whom had previous experience in this field. They were under the direct supervision of the assistant technical director—the only engineer present at all the tests, who followed through all the operations. These assistants were picked with the cooperation of WPA officials, and were then taught to read the various instruments. All required computations were reduced to such simple forms that it was possible to have practically all operations, both in the field and in the office, performed by this staff, despite the fact that its members were only unskilled workers.

Because of the large number of points to be read at each increment of load, the load was applied to a predetermined value, and then held for a sufficient period to read the various gages. Loading was then resumed, and at the next predetermined load point the process was repeated. These reading intervals lasted from a few minutes or less to a half hour or more, depending on the number of gages and other factors. Errors in reading occasionally occurred, because of the inexperience of the workers. When discrepancies were found in the final analysis, the more logical reading was taken or the questionable reading was not used in any computations. In still other cases, the questionable reading has been given in the detailed report with a note of caution indicating the possibility of error.

In timber testing, a variation in results is usually introduced by changing the speed by which a specimen is loaded. Because of the peculiar equipment used in this program, it was not possible to employ a uniform speed; nor were the rates of loading specifically determined. Since a structure is seldom, if ever, loaded at a uniform speed, the value of the test results should not be discounted because of the methods used. In all cases, the times of reading and loading were recorded; and, in this report, the total elapsed time is indicated on the individual test.

Truss Tests.—Trusses were mounted on the frame with their longitudinal planes horizontal—in other words, in a flat position. They were suspended from the roof by hangers composed of two planks, 2 in. by 8 in., nailed to form a T-section. The web of each T-section was nailed to the roof rafters overhead and the flange was nailed to the truss. These roof rafters were then tied down to the frame of the building with plumber's tape, so that the hangers also acted

as struts to prevent members of the truss from buckling upward under load. The truss was thus supported at varying spacings, usually from about 4 ft to 8 ft—depending on its weight, dimensions of top chord, locations of gages, and other factors. These hangers offered practically no resistance to the horizontal deflections of the truss specimens under load, for they could be moved easily by hand.

Truss loads were applied in varying increments up to the predetermined point at which the dial gages were removed. Loads were then increased until failure occurred, only the deflection and micrometer gages being read. On several trusses, failure occurred earlier than expected, while the gages were still in place. Reactions were taken on blocks that simulated the actual restraining conditions in the structure from which the truss was removed.

Joint Tests.—The testing frame was designed with the thought of testing full-sized joints of structural frames as economically as possible. In the past,

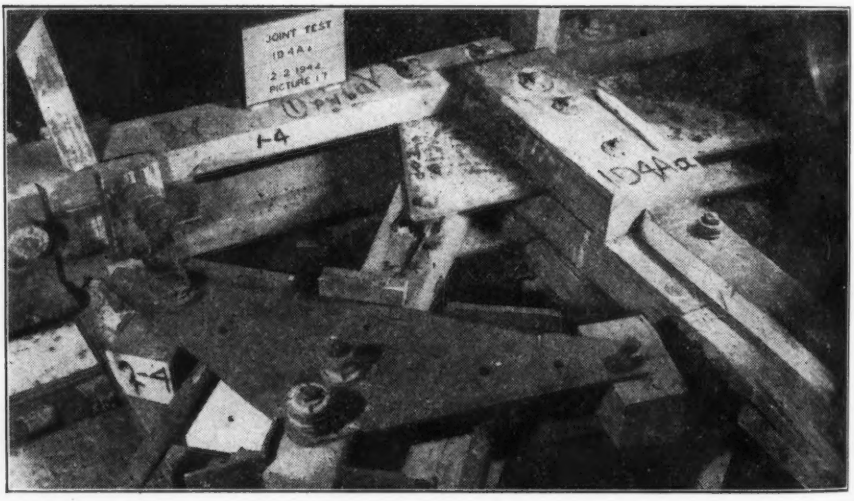


FIG. 1.—TYPICAL FAILURE OF TENSION DIAGONALS OF SHEAR PLATED JOINTS (JOINT J5)

it has sometimes been the custom to test two identical joints at a time, back to back. This method was not considered desirable nor practical in this program; and, in those joints which consisted of three or more members meeting at a common intersection, a procedure was used as shown in Fig. 1. The load was applied through the hydraulic jack, putting compression in one of the members, and was resisted by another member in compression, blocked tightly against the testing frame. In some specimens, a screw jack was installed on this second compression member to help adjust the specimen and to reduce possible secondary stresses. The third member necessary to preserve equilibrium was put in tension by the following procedure. Before testing, large steel plates were fastened to it with bolts or shear plates to provide the necessary tension connection; and, after placing the joint in the testing frame, these plates were connected to two 2-in. steel rods, which were in turn fastened to

the testing frame. One exception to this procedure was joint J13 in which all the members were put in compression. Gages were installed to measure the various deformations. In almost all the joints tested, additional gages were installed to measure the angular change between members, as a check on any possible secondary stresses that might be induced by the deformation of the joint. In no case were the movements such that these secondary stresses influenced the tests.

Plywood Tests.—The tests of the strength of nails in plywood in resistance to lateral forces, and of plywood in resistance to shear and buckling, were made in the testing frame, as described later in Section D–IV.

Reduction of Data and Calculations.—After the field data were typed and mimeographed, they were reduced, calculations were made as necessary, and stress-strain curves were plotted. Stresses in members of trusses and joints were calculated, making use of forms and with the usual assumptions for the actions of structures under load. In the case of a truss with eccentric joints, stresses were first calculated by assuming pin-ended concentric connections at the panel working points. Then corrections were made, as necessary, to the stress in any members for change of slope and the increased shear in the panel caused by eccentric connection. Stresses in these connections were calculated on an approximate basis only, as a more theoretically correct analysis would have involved unnecessarily laborious computations on structures statically indeterminate to many degrees.

One feature of the mechanics of recording the results of the tests was the use of photography. All specimens were photographed before and during the tests, at failure, and at the disassembly of the specimen to record all possible visual features. These photographs and their accompanying notes form an indispensable record. In the event that other investigators review this work, the field notes should be used with the photographs. For the information of those interested, the data observed, with the curves, field notes, photographs, calculated reductions, etc., were filed at the San Francisco office of the National Lumber Manufacturers' Association, Engineering Societies Library in New York, N.Y., and the Forest Products Laboratory in Madison, Wis. This report includes only a small percentage of the total data available.

II. Safety Factor

One of the principal results expected from these tests was a ratio between loads causing failure in the specimens tested and the calculated allowable load that the specimen, in its weakest part, might be expected to sustain safely. Such ratios, customarily designated as "factors of safety" are likely to be misleading in a composite structure if compared to factors of safety of isolated test specimens. Quite different results may be expected, for instance, when a joint is tested (1) as an isolated specimen in a testing machine, (2) as a part of a truss tested by itself, or (3) as a part of a roof assembly in a building where the assembly as a whole may provide additional stability. Furthermore, factors of safety used in determining safe working load for nails, bolts, and connectors and for timber are derived from different considerations (ultimate load, proportional limit, deformation, etc.) and hence are not properly comparable.

For these reasons the term "factor of safety" has not been used in this report. Instead a ratio, designated by R, is used in presenting test results. In discussing the tests, factors R are given, which are ratios between the computed member stress at ultimate loads and the usually accepted design capacities (allowable loads), which are further defined in each test or group of tests. These allowable loads, where possible, are based on the recommendations of the Forest Products Laboratory or those of the Timber Engineering Company, Inc. (TECO), in its 1939 manual.[1]

It should be emphasized that, in most tests, failure occurred in a manner neither expected nor previously investigated and the factors R are based on the nearest available criterion usually used by designers. This criterion is clearly defined in each case. For example, in one truss of the North Square Building, the factor R is based on the allowable connector capacities, although failure was clearly due to tension perpendicular to the grain.

III. Nomenclature

Notation.—The following letter symbols, used in this report, and in its discussions, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932) prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = section area; in Table 26, the symbols A to Q are introduced as key letters for typical dimensions indicated in Figs. 84 and 85;
d = depth of a beam;
E = modulus of elasticity in tension and compression;
e = eccentric distances;
F = total stress in a member, imposed by external loads; F_A = total stress in member A, etc.;
K = a factor derived from the curves of Fig. 83(*c*);
L = length of span or height of a column; height of a panel;
P = an applied load:
- P_b = allowable load on bolts;
- P_c = allowable load on connectors;
- P_l = proportional limit load;
- P_{bl} = proportional limit load on bolts;
- P_{cl} = proportional limit load on connectors;
- P_{wa} = allowable load on wood;

R = ratio between the computed stress at test loads and the usually accepted design capacities (allowable loads):
- R_{ba} = ratio R between maximum test load and allowable load on bolts;
- R_{bl} = ratio R between proportional limit load and allowable load on bolts;
- R_{ca} = ratio R between maximum test load and allowable load on connectors;

[1] "Timber Connector Construction," Manual of the Timber Eng. Co., Inc., Washington, D. C., 1939.

R_{cl} = ratio R between proportional limit load and allowable load on connectors;
R_{wa} = ratio R between maximum test load and allowable load on wood;
R_{wl} = ratio R between proportional limit load and allowable load on wood;
R_l = ratio between computed stress at the proportional limit loads and the usually accepted design capacities (allowable loads);
r = least radius of gyration;
s = stress per unit area; s_{wa} = allowable unit stress in wood;
W = total applied load on truss;
w = applied load on truss, in pounds per linear foot;
X = an unknown total force;
Δ = total deformation or displacement; and
θ = angle of twist; also the slope of the grain in relation to the axis of a member.

Explanation of Terms and Abbreviations.—

1. Allowable Stresses in Timber.—The allowable load W on a structural member of either one or more pieces, when determined by the strength of the timber, using the recommended unit working stresses s of the Forest Products Laboratory is termed "wood allowable load," and is abbreviated P_{wa}. The unit working stresses for selected structural Douglas fir (the grade used in practically all cases) are:

Description	Stress, s_{wa} (lb per sq in.)
Bending or tension parallel to the grain	1,600
Longitudinal Shear Stress:	
In beams	100
In joints	150
Compressive Stress:	
With the grain	1,200
Across the grain	350

On surfaces inclined at an angle θ with the direction of the grain, the allowable working stress s_{wa} in compression is expressed by the Hankinson formula as given subsequently in Eq. 1.

2. Allowable Stresses on Bolts.—The allowable load on a structural member of either one or more pieces when determined by the safe load on the bolts in a bolted connection, as computed in accordance with the recommendations of the Forest Products Laboratory,[2] is hereinafter termed "bolt allowable load" and is abbreviated P_b.

3. Allowable Loads on Connectors.—The allowable load on a structural member, of either one or more pieces, determined by the recommended standard design loads given by TECO,[1] is hereinafter termed "connector allowable load," and is abbreviated P_c.

[1] "Wood Handbook," Forest Products Laboratory, Forest Service, U.S.D.A., Madison, Wis., 1940.

4. Compression Diagonals.—When used in the description of a joint test, the designations "compression diagonal" and "tension diagonal" refer to the nature of stress in the diagonal during the test, and not necessarily to stress function in the structure from which it was taken.

5. Proportional Limit.—The load at the proportional limit, as judged by the load-deformation curves, is termed hereinafter the proportional limit load and is abbreviated P_l.

6. Maximum Stress Ratio.—The ratio of the maximum stress in a member or connection, to a calculated allowable load is hereinafter termed R_{wa}, R_{ba}, and R_{ca}, respectively.

7. Proportional Limit Ratio.—In a manner similar to item 6, the ratios of the proportional limit load to the allowable loads are R_{wl}, R_{bl}, and R_{cl}, respectively.

8. Malleable Iron Washers.—The designation MIW means "malleable iron washer"; for $\frac{7}{8}$-in. bolts, $3\frac{1}{2}$ in. in diameter and $\frac{7}{16}$-in. thick; and, for $\frac{3}{4}$-in. bolts, 3 in. in diameter and $\frac{7}{16}$-in. thick.

9. Identification of Structures.—The Exposition buildings from which the trusses tested were taken, and their abbreviations when applied to trusses, are identified in this report as follows:

Building	Abbreviation
Alameda-Contra Costa Counties Building	AC
Brazil Building	B
Cavalcade	C
Chinese Building	Ch
French Building	F
Merry-Go-Round	M
North-Square Court	NS
San Francisco Building	SF

10. Connectors.—Split rings or shear plates, used for connecting timbers in various ways, are termed "connectors."

11. Member.—A structural element that extends between two panel points of a truss or structural frame and acts primarily in resistance to direct or axial tension or compression is a "member." The term includes those structural elements of test specimens which had so functioned in the structural frames from which they were taken. A member consisting of one piece or one "stick" of solid cross section—as a member composed of one 6-in. by 10-in. section—is variously termed: "One-piece," "one stick," or "single member." Similarly, a member consisting of two separate pieces or two separate "sticks"—as a member composed of two 3-in. by 10-in. pieces—is variously termed: "Two-piece," "two stick," or "double" member.

12. Single Shear.—A member has its end connection, or connections, in single shear when its connectors are in one face of the member only, or when bolts or nails attach the member to a connecting member, splice pad, or gusset plate by only one face.

13. Double Shear.—A member has its end connection, or connections, in double shear when its connectors are in two opposite faces of the member and

are equally divided in number between each face; or when bolts or nails attach the member to a connecting member, splice pads, or gusset plates by two opposite faces.

14. Loose Bolts.—Examination of the bolting of the specimens of this test program led to the conclusion that all bolts had been inserted in slightly oversized holes. Nevertheless, the bolts were found to be tight in their assemblies—that is, they could not be turned by hand. Shrinkage of the wood, from its green condition at the time of framing to its seasoned condition at the time of testing, had caused the sides of each wood member, through which a bolt passed, to draw away from the washers, or from the gusset plate, or plates, to which the member was connected. This, then, is the meaning of such phrases and terms as "the bolts were loose," and "loose bolts."

15. Wood Handbook.—Wood Handbook refers to the "Wood Handbook"[2] prepared by the Forest Products Laboratory of the United States Department of Agriculture.

D. DETAILED REPORT OF TESTS

I. Truss Tests

Alameda-Contra Costa Counties Building (AC) *Trusses.*—For purposes of identification the truss numbers of this building are prefixed by the letters TAC.

These tests were of four trusses from the AC Building, with compression diagonals and tension rods and, except for minor variations in depth of daps and cuts, of identical construction. They were 39 ft 4½ in., center to center of span, about 6 ft deep, and had 6-in. by 8-in. top and bottom chords substantially parallel. Fig. 2 shows the detail. Except for truss TAC–3 the trusses were in good condition. This truss, when tested, was found to have been injured either during erecting or dismantling.

Workmanship appeared to have been good; daps and cuts were well fitted. Bolts at the splices were generally tight in their holes, although spaces between truss members and splice pad indicated that shrinkage of from about ⅛ in. to ¼ in. had occurred. Some of the washers were loose, but the bolts themselves could not be moved by hand. Timbers consisted of Douglas fir of select structural grade, and the trusses were tested with an average moisture content of 12%. The end butt blocks at joints L1 and L7 were in good condition, and no checks were observed in the horizontal plane of theoretical maximum shear—for example, on plane A1-A2, Fig. 3. A sound knot was found in this plane in truss TAC–4.

Testing.—As usual, trusses were tested in a horizontal position, the loads being applied as concentrations at the upper panel point as shown in Fig. 2. Deformations, particularly those of the end joints, were measured by gages. Referring to Fig. 4, a typical installation at one end joint: Gage A measures the horizontal slip between block and chord; the stick between points X and Y is fastened to butt blocks by using blocks at the ends, so that gage B gives a measure of the bending deformations of the block as a beam between points X and Y as well as of the butt blocks; gage C measures movements of the projected end of the diagonal perpendicular to its axis—the movement being

measured with respect to the bottom chord at the working point; gage D measures deformations parallel to the diagonal, between points D and Z as the stick between these two points is fastened to the diagonal only, thus projecting movements to gages C and D; gages E, F, and G measure deformations

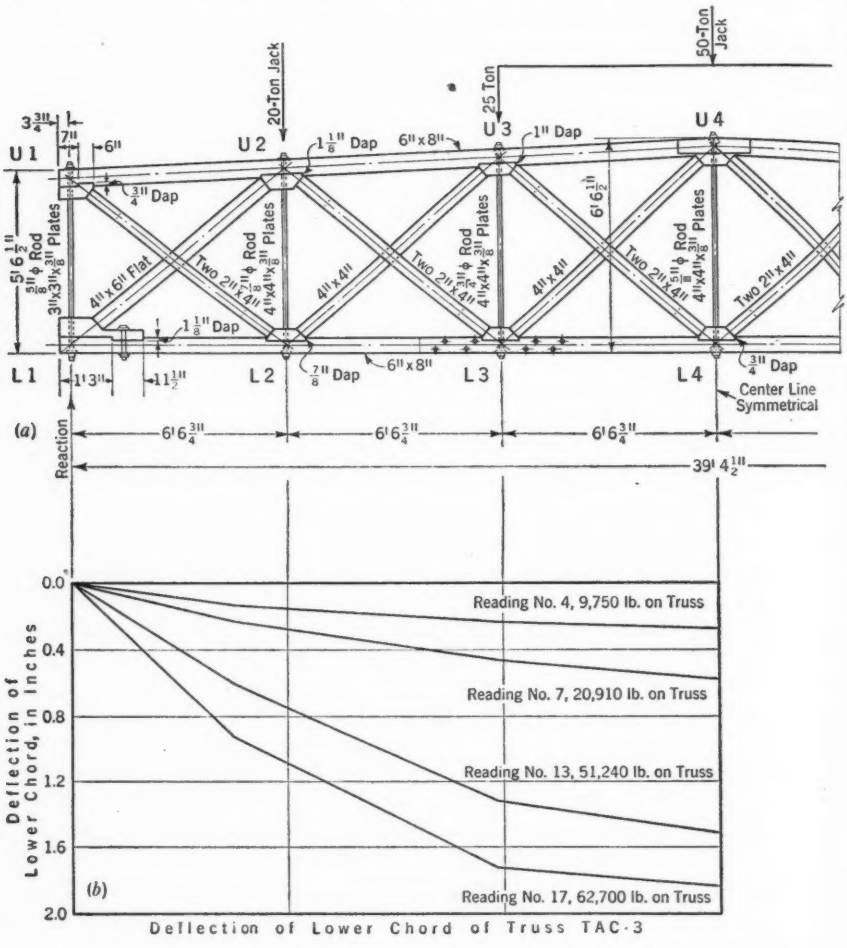


FIG. 2.—DETAILS OF TRUSS TAC-1; TRUSSES TAC-2, TAC-3, AND TAC-4 ARE SIMILAR

of the block perpendicular to the chord at the places shown; and gage H measures the "bedding" of the end of the diagonal into the bevel cut of the block. Similar gages were installed on both faces of the joints.

In two trusses, TAC–3 and TAC–4, a smooth steel plate, which had been rubbed on both sides with paraffin, was installed at an end joint to reduce friction between chord and block.

Truss TAC–1.—The first evidence of overstress in truss TAC–1 was observed at joints L7 and U2 at the butt blocks, where the end diagonals were

bearing on their beveled cuts. At joint U2, the block showed small splits and about ½-in. embedment of the diagonal. The load in member L_1U_2 at this

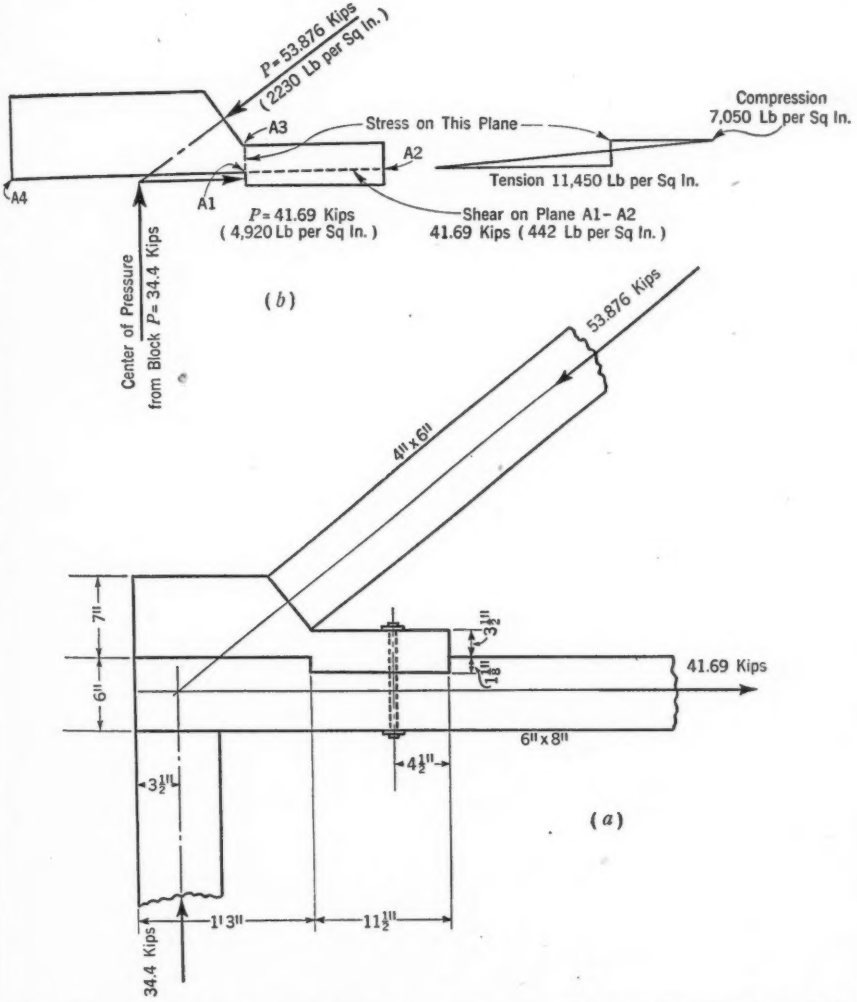


FIG. 3.—APPROXIMATE CALCULATED STRESSES AT JOINT L7, TRUSSES TAC-1, TAC-2, TAC-3, AND TAC-4

time was about 45 kips, giving a bearing stress on the butt block of about 2,000 lb per sq in., although this indentation of ½ in. did not cause the load on the truss to decrease. Final failure occurred very suddenly in shearing off the table of the butt block at joint L7, as shown in Fig. 5. The shearing plane was A1-A2, Fig. 3. The load in member U6–L7 was 49 kips, and the calculated shearing stress was 474 lb per sq in., neglecting friction.

Truss TAC–2.—Testing on this truss was begun in the afternoon and continued until evening when loads were released, and was resumed the following

morning, when failure occurred, as in truss TAC–1, at joint L7 in shearing off the table of the end butt block. Before failure, as shown by the arrow in Fig. 6, a space opened between the inner end of the butt block and the chord,

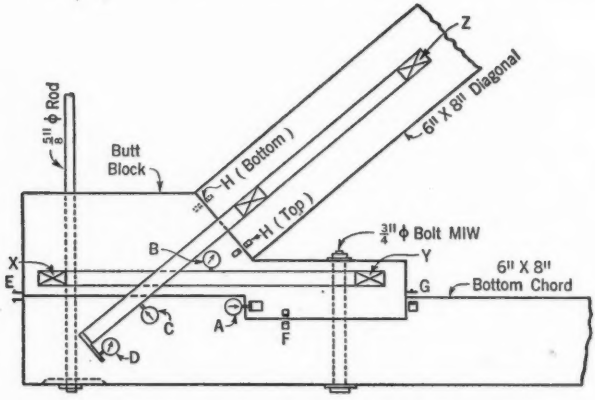


FIG. 4.—TYPICAL INSTALLATION OF GAGES AT AN END JOINT

caused by the eccentric pull of the table of the butt block. This phenomenon was typical for all four trusses, although not to the same degree. A detailed analysis of the moments and stresses involved is shown in Fig. 3.

Failure did not occur immediately upon reaching the maximum load, but about 2 min later.

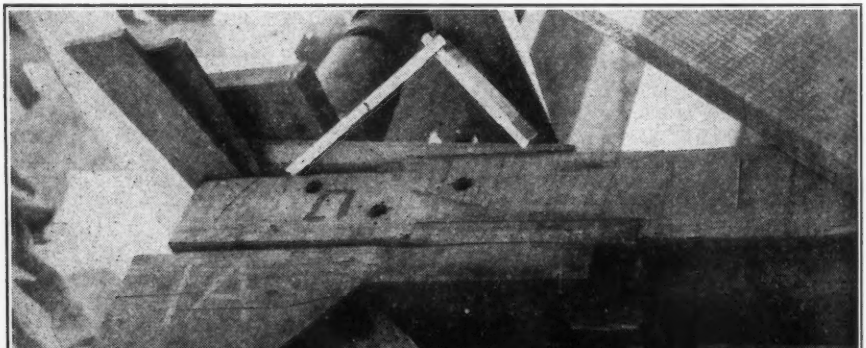


FIG. 5.—FAILURE OF BUTT BLOCK AT AN END JOINT

Truss TAC–3.—As previously noted, a paraffined steel plate was installed at the end joint of panel point L7 in truss TAC–3. Loading was started on a

Saturday; and, at a total load of 49.7 kips on the truss, the lower chord failed in tension between two panel points. The load was released and examination indicated the chord had suffered a previous injury. The chord was repaired by splicing. On the following Monday afternoon testing was resumed, when

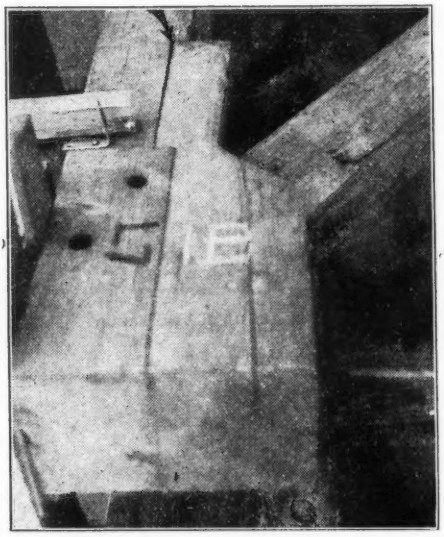


FIG. 6.—BENDING OF BUTT BLOCK PREVIOUS TO FAILURE IN SHEAR

failure occurred at joint L7 after the maximum load had been applied for about 6 min. The failure was similar to those described for trusses TAC–1 and TAC–2.

Truss TAC–4.—A paraffined plate was also installed in the end joint of panel point L1 in truss TAC–4. Failure occurred at this joint about 5 min after the maximum load had been applied in a manner similar to that previously described.

Results of Tests, Trusses TAC.—Results of these tests are listed in Table 1. Fig. 2(*b*) includes the deflection curves for the lower chord of one of the trusses, for four different loadings. Fig. 7 shows center deflection of the upper and lower chords of truss TAC–1. Fig. 8 shows the opening of the lower chord splices at L3 and L5; Fig. 9 shows the embedment of the end diagonals in the bevel cuts of blocks at U6; Fig. 10 shows the deformation at gage D; Fig. 11 shows the slip of butt blocks with respect to the chord at gage A. Six curves are presented in Fig. 12, showing deformations at gages on butt blocks of joints L1 and L7 on truss TAC–3, the location of the gages being shown in Fig. 4.

Computations based on a unit shear stress of 150 lb per sq in. in the end butt block indicate that the trusses were designed for an allowable load of about 660 lb per lin ft. Trusses TAC–1 and TAC–2 failed at $R_{wa} = 3.15$ and 2.95, respectively. The two trusses on which paraffined steel plates were installed failed at $R_{wa} = 2.55$ and 2.0.

TABLE 1.—SUMMARY OF TEST RESULTS, STRESSES IN THE TRUSSES OF THE ALAMEDA-CONTRA COSTA BUILDING (AC)

Truss	Center Deflection			Truss failed at panel point (Fig. 2)	Maximum load on truss[c] (kips)	Shearing Stress s (lb per sq in.) on Block That:		Ratio R_{wa} [d]	Total Time Elapsed		Deflection of end blocks at allowable load[e]
	Under load of 600 lb per lin ft[a]	At Loads of Nearly:[b]				Failed	Did not fail		Hr	Min	
		Load (kips)	Deflection								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
TAC–1..	1/790	62.70	1/255	L7	68.55	474	510	3.15	3	50	1/1,030
TAC–2..	1/665	60.73	1/257	L7	68.55	443	430	2.95	2	35[f]	1/980
TAC–3[g]..	1/774	45.25	1/378	L7[h]	58.82	382	385	2.55	2	55[f]	1/7,200
TAC–4[g]..	1/755	37.85	1/468	L7[h]	45.25	297	326	2.00	2	10	1/4,600

[a] Center deflections (expressed in fractions of the span length) at a computed allowable truss load of 600 lb per lin ft, measured on lower chords. [b] Center deflections in Col. 3 (expressed in fractions of the span length) at ultimate loads near (not at) the loads given in Col. 2. [c] Concentrated loads applied at panel points U3 to U6, inclusive. [d] Based on an allowable shearing stress s_{wa} of 150 lb per sq in. [e] Deflections of end blocks (expressed in fractions of the span X and Y), at allowable loads, under secondary bending. [f] On two days. [g] These trusses had paraffined steel plates in one end joint. [h] Failed at end joint with paraffined steel plates.

TABLE 1.—(*Continued*)

Truss	Deflection of End Blocks Under Secondary Bending[i]		Stresses, in Pounds per Square Inch						Average Ultimate Shears in End Blocks That Failed	
			Computed Maximum in the Block							
			Direct Tension[j]		Bending		Total[k]			
	Load (kips)	Deflection	At s_{wa}	At ultimate load	At s_{wa}	At ultimate load	At s_{wa}	At ultimate load	Lb per sq in.	R_{wa}
	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
TAC–1..	32.04	1/670	750	2,340	3,260	10,200	4,010	12,540	457[l]	3.1
TAC–2..	46.26	1/555	740	2,320	3,280	10,300	4,020	12,260	....	....
TAC–3..	45.25	1/1,220	700	1,890	3,310	8,910	4,010	10,800	340[k, l]	2.25
TAC–4..	37.85	1/930	740	1,550	3,590	7,500	4,330	9,050	....	....

[i] Expressed as fractions of the span between points X and Y, Fig. 4. Deflections in Col. 13 are the highest readings when the load on the truss was as given in Col. 12. [j] Parallel to the grain. [k] Computed maximum tension plus bending stress in the block at the plane A_1–A_2. [l] The difference between these two stresses (117 lb per sq in.) corresponds to a coefficient of friction of 0.315.

Loads at failure of the tables of the butt blocks (at plane A1-A2, Fig. 3) produced calculated shearing stresses of 297 lb per sq in., 382 lb per sq in., 443 lb per sq in., and 474 lb per sq in. The first two values of s_s are for the two trusses in which paraffined steel plates were used, the average shearing stress being 340 lb per sq in. Where plates were not installed, the average ultimate shearing stress was 458 lb per sq in. This increased strength, amounting to 35%, may be explained by the frictional resistance on plane A1-A4 in trusses TAC–1 and TAC–2, relieving the shearing stresses on plane A1-A2.

Failures in the blocks at these relatively low unit stresses were probably due to the abrupt change in section at point A1, resulting in a combination of

high bending and shearing stresses occurring simultaneously in the vertical plane at point A1.

Fig. 3 presents an analysis of the stresses in the end butt block of truss TAC–2 at the time of failure. In the analyses it has been assumed that the bolt holding down the block was loose and therefore did not act in tension to reduce the bending moments in the vertical plane at point A1. Actually, the washers under the heads of these bolts were so loose that they did not become

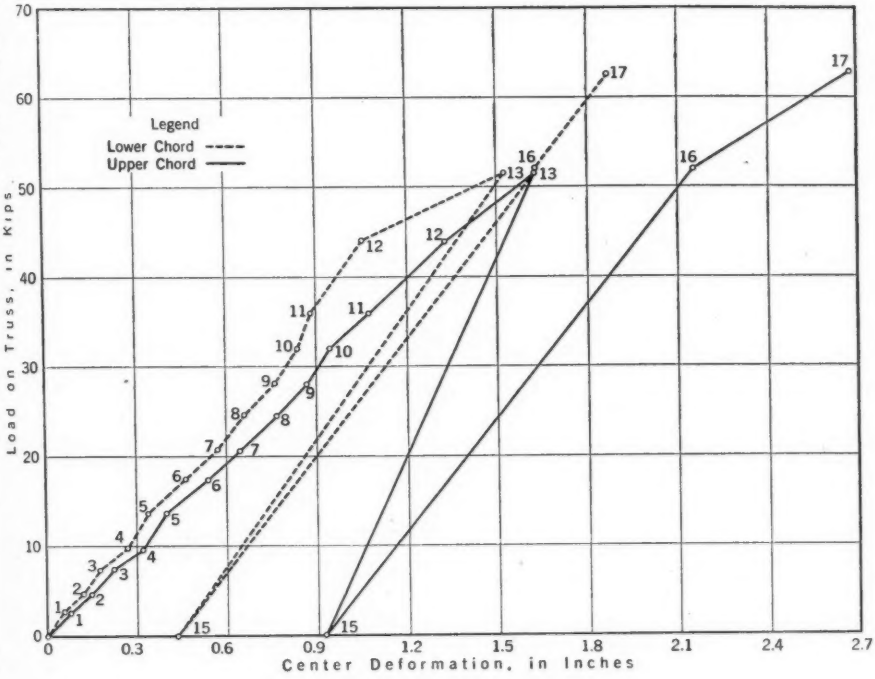


FIG. 7.—CENTER DEFLECTION, TRUSS TAC–1

tight even in the tests. The bolts could not be moved by hand, however, so that some slight tension may have been present through friction on the surfaces of the bolts. If any such tension did exist, it would have resulted in somewhat reduced bending stresses in the butt block.

The tests of the two trusses in which the end joints were modified by the introduction of paraffined steel plates seem to indicate that friction plays an important part in the resistance of this type of end joint. However, the tests are not sufficient in number to justify reliance on friction in timber design.

The tests also confirm what good judgment would have dictated—in the end butt blocks, top and bottom cuts should not be made in the same vertical plane. They also seem to show that abrupt changes in cross section of wood members reduce the strength more than is accounted for by present design practice and that, when such abrupt changes cannot be avoided, lower unit working stresses should be used. Further tests on joints in which a combina-

tion of shearing stress, direct stress, and bending stress is involved are much needed for clarification.

The effect of shrinkage causing increased stresses in timber joints is illustrated in these tests where the bolts in butt blocks became ineffective in tension, thus introducing bending and causing local flexural stresses.

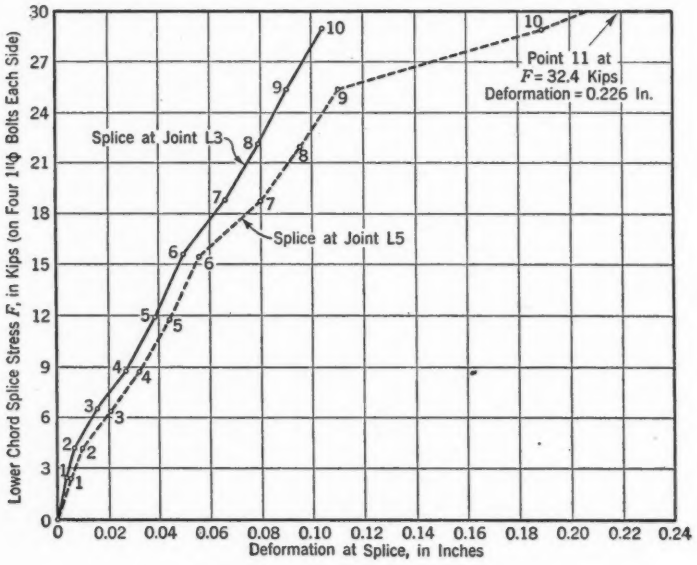

Lower Chord Splice Stress F, in Kips (on Four 1¼ Bolts Each Side)
Deformation at Splice, in Inches
Splice at Joint L3
Splice at Joint L5
Point 11 at
F = 32.4 Kips
Deformation = 0.226 In.

FIG. 8.—OPENING OF SPLICES, JOINTS L3 AND L5

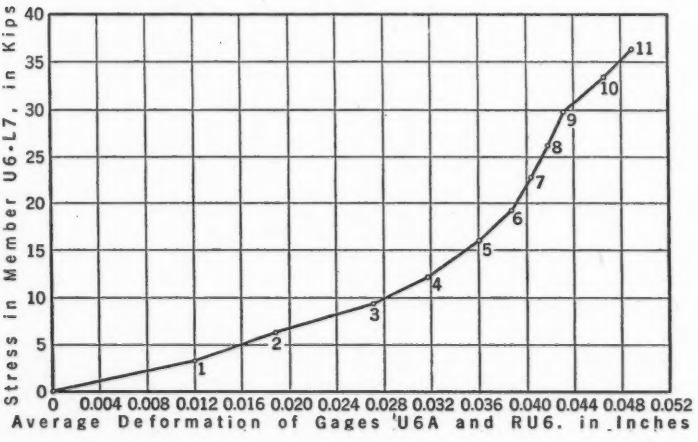


FIG. 9.—EMBEDMENT, AT THE END DIAGONAL, INTO THE BLOCK AT JOINT U6

The allowable tension in the lower chord splice at panel point L3, as determined by the four 1-in. bolts in one half of the splice, is 16,800 lb. The tension in the splice at the average ultimate load on the truss (representing

failure of the end butt blocks in shear and not failure of the splice) was 60,000 lb. In the splice bolts, therefore, R_{ba} would have a value of 3.6; and, since the splice evidenced no weakness, the potential value of R_{wa} or R_{ba} would be greater. In trusses TAC–1 and TAC–2, which sustained the maximum ultimate loads (68,500 lb), the value of R_{ba} would be 4.1.

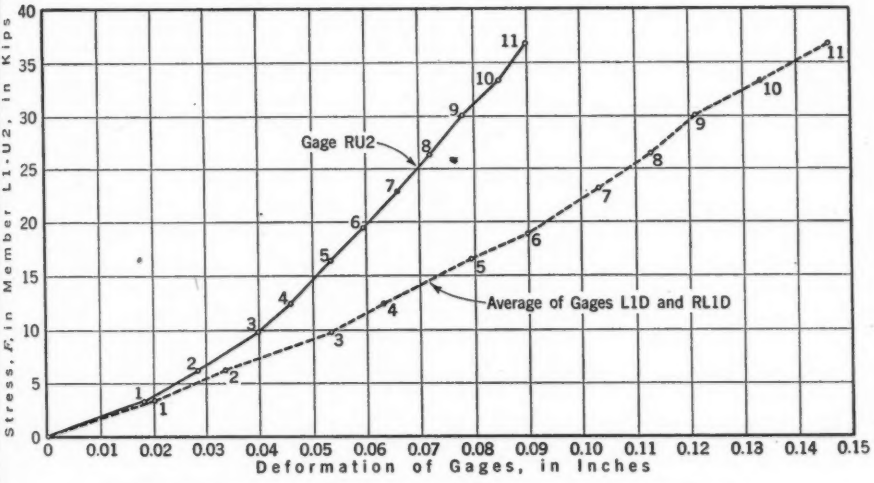


FIG. 10.—TOTAL AXIAL DEFORMATION OF DIAGONAL WITH RESPECT TO THE CHORDS AT PANEL POINTS L1 AND U2

This series of tests furnishes another illustration of the need for tests of full-sized timber trusses and structures since only in such way can combinations of different kinds of stresses show their true effect.

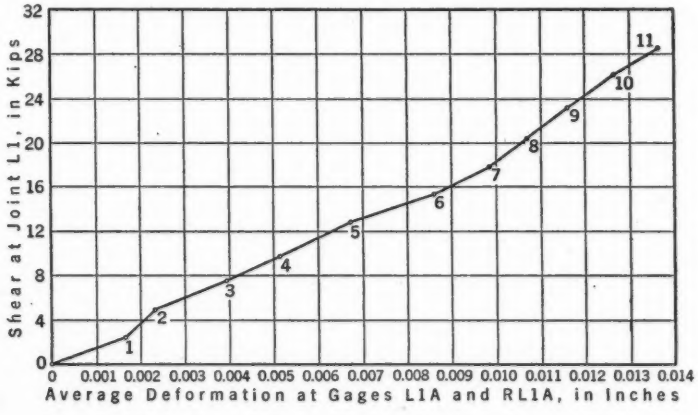


FIG. 11.—SLIP OF BUTT BLOCKS WITH RESPECT TO THE CHORD AT GAGE A

Brazil Building (B) *Trusses.*—Two shallow trusses (perhaps, more accurately, two trussed frames) about 46.5 ft long and averaging 2 ft 10 in. deep were obtained from the Brazil Building and tested. Originally they formed

the vertical legs of a rigid-frame bent. Unfortunately, the building had been dismantled carelessly; and, although the two frames, when carefully examined, appeared to be in good condition, the possibility exists that they had been damaged. No evidence of such damage, however, was disclosed by the tests.

The chords of the frames were formed of three 12-in. planks each, with the web members of two 3-in. by 6-in. timbers, connected to the chords by split rings and bolts. The outside 2-in. by 12-in. planks of the chords were quite

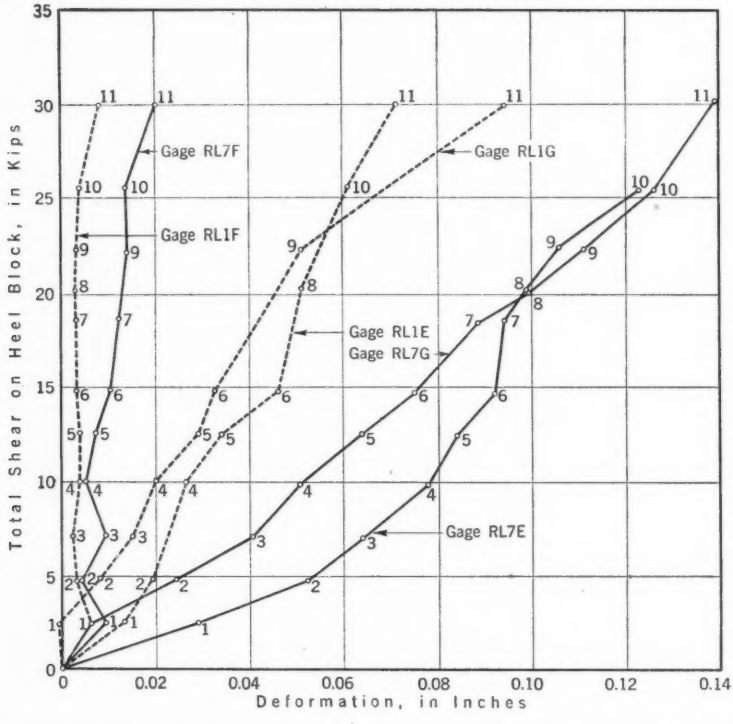


FIG. 12.—DEFORMATIONS AT VARIOUS GAGES ON THE BUTT BLOCKS OF JOINTS L1 AND L7, TRUSS TAC-3

badly checked, especially at points where two lines of bolts and rings were used. The timber was a select structural grade of Douglas fir and was tested at a moisture content of about 15%.

Details of one of the frames are shown in Fig. 13, the other frame being practically identical. The connections of both frames were eccentric—that is, the center lines of connected members at a joint did not meet at a common point.

Testing.—The frame had its loads applied as shown diagrammatically in Fig. 13, in which the points of application of the loads were changed during the test. Deflections and joint deformations were measured as usual; and, in addition, gages were installed on one chord to measure the relative movement of the center plank with respect to the outside plank.

Trouble was experienced in testing both frames because of the large deformations. The loading jacks had a maximum extension of 6 in. so that considerable loading, unloading, and reblocking were necessary to obtain failure of the frames. Thus, testing of frame TB–1 consumed 3 hours, and frame TB–2 required 1 hour and 10 min.

In the test frame TB–1 the first failure occurred at joint U1 and almost simultaneously at joint L10, in tension perpendicular to the grain. About the same time, failure occurred in the lower chord splices. Upon dismantling the truss, it was found that the split rings were missing in one each of the vertical web members of joints L7, L8, and L9, although grooves for the rings had been provided. As failure did not occur in these web members, these conditions apparently did not affect the load capacity of the frame.

In frame TB–2, the tension splices of the lower chord started to open at small loads. When a load of 17.2 kips on the frame had been reached, the center 3-in. by 12-in. plank of the lower chord broke near joint L5 (see Fig. 14). The increase of the load to 23.5 kips sheared the wood in the end distances in the lower chord splices as shown in Fig. 13.

Results of these tests are summarized in Table 2 and deflections for frame TB–1 for five loadings are shown in Fig. 13. Fig. 15 indicates the center deflection for the same frame.

TABLE 2.—SUMMARY OF TEST RESULTS; STRESSES ON THE TRUSSED FRAMES OF THE BRAZIL BUILDING (B)

Frame	Deflections					Ultimate load (kips)	Splice load[b] (kips)	R_{ca}	
	At a Load of 10 Kips		At Other Loads, P					1939	Splice[c]
	Inches	Ratio[a]	P	Inches	Ratio[a]				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
TB–1.....	2.3	1/233	26.8	5.0	1/107	30.1	117.8	2.95	3.5
TB–2.....	2.5	1/214	22.0	5.8	1/ 93	17.2[d]	99.8	2.48	3.0

[a] Ratio of span. [b] Load on a splice at failure, computed by the ordinary truss theory. [c] The ratio of R_{ca} for the connection of members U1–L2 and U9–L10. [d] In frame TB–2 the center stick (3 in. by 12 in.) broke in bending at a load, on the frame, of 17.2 kips. The splices failed completely when the load on the frame was increased to 23.5 kips.

Fig. 16 shows the relative movements of the three parts of the lower chord under load. The location of the gages for these points is shown in Fig. 13.

Results of Tests, Trusses TB.—The action of these frames under load is complicated and analysis of test results, therefore, is difficult. Neglecting the bending effects of the chords and using direct stresses as computed by the usual truss theory, the values of R_{ca} calculated for the splices are 2.95 and 2.48. If the bending moments in the two chords, due to deformation of the truss as a whole, are considered, these ratios would be changed.

The importance of stresses due to deflection in shallow trusses, frames, or girders is illustrated in the failure of frame TB–2 in which the through, 3-in. by 12-in. stick of the chord member broke in bending, mainly as the result of

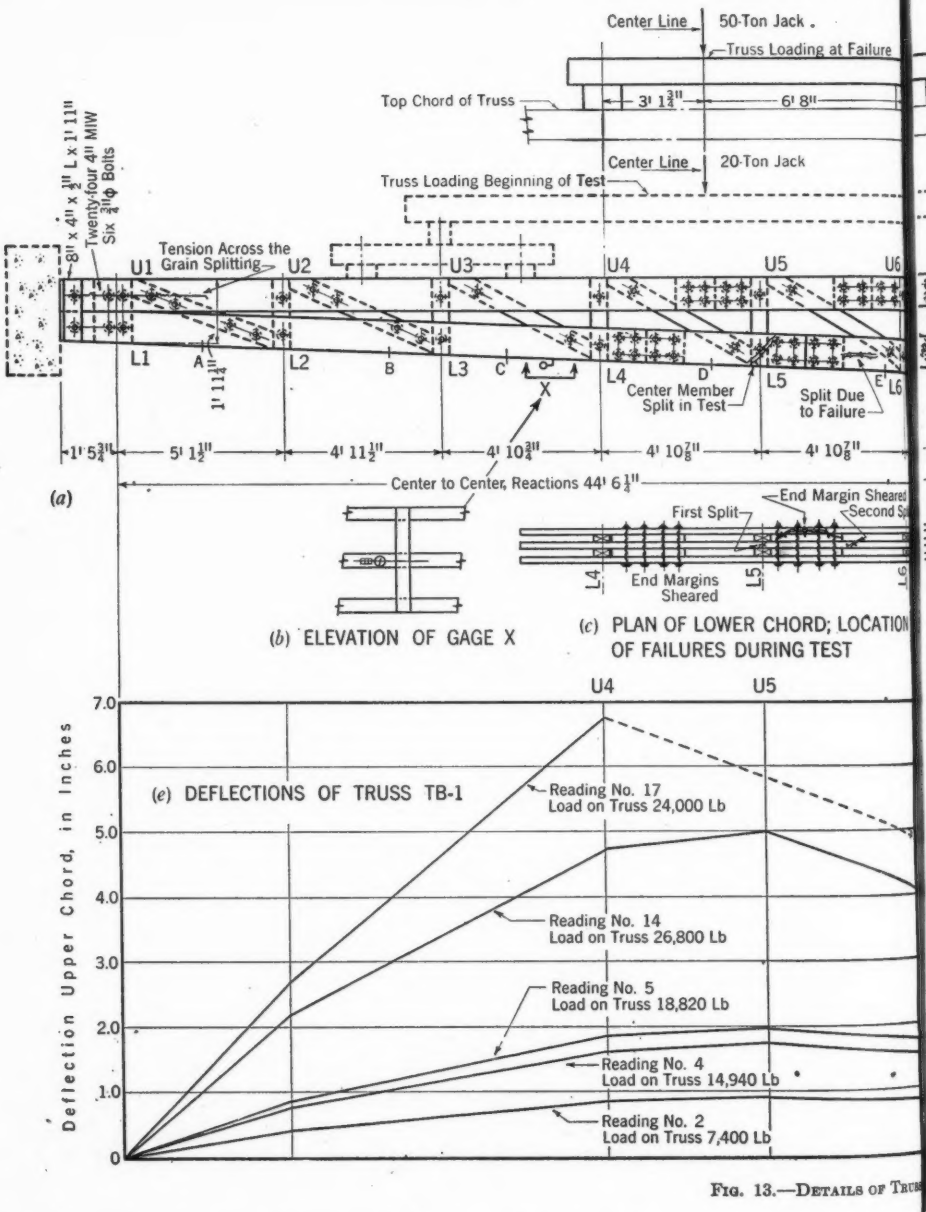
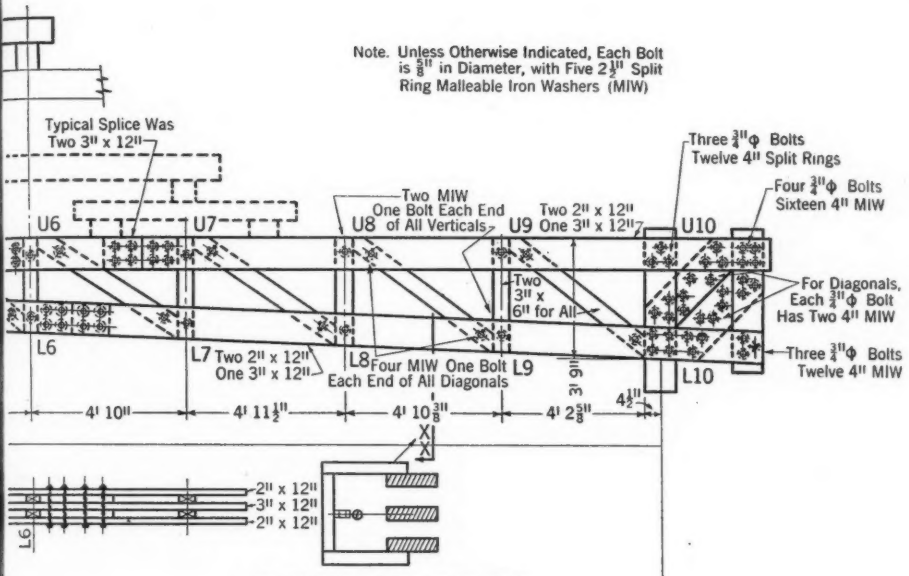


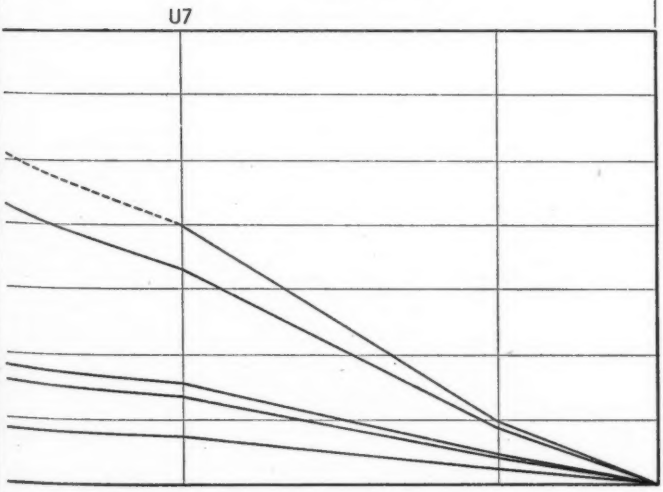
FIG. 13.—DETAILS OF TRUS

deformations in the adjacent splices of the 2-in. by 12-in. sticks, and of deformations of the frame as a whole.

At the two ends of frame TB–1, failures occurred simultaneously in tension perpendicular to the grain of the chord members. Failure in tension perpendicular to the grain at eccentrically connected joints was illustrated in this


Note. Unless Otherwise Indicated, Each Bolt is ⅞" in Diameter, with Five 2½" Split Ring Malleable Iron Washers (MIW)
Typical Splice Was Two 3" x 12"
Three ¾"Φ Bolts Twelve 4" Split Rings
Four ¾"Φ Bolts Sixteen 4" MIW
Two MIW One Bolt Each End of All Verticals
Two 2" x 12" One 3" x 12"
U6
U7
U8
U9
U10
Two 3" x 6" for All
For Diagonals, Each ¾"Φ Bolt Has Two 4" MIW
L6
L7 Two 2" x 12" One 3" x 12"
L8 Four MIW One Bolt Each End of All Diagonals
L9
L10
Three ¾"Φ Bolts Twelve 4" MIW
4' 10"
4' 11½"
4' 10⅜"
4' 2⅝"
4½"
2" x 12"
3" x 12"
2" x 12"
X
X

(*d*) GAGES A TO G INCLUSIVE
SECTION *X-X*



TB-1; TB-2 Is Similar

test as well as in other tests of this program, in which the members of a joint were connected eccentrically. The nature of the failures emphasizes the importance of the consideration of secondary stresses in main members, splices, and connections in shallow trusses which may be expected to show relatively large deflections and deformations at working loads.

Cavalcade (C) *Cantilever Truss.*—Opportunity was afforded to test, in place, one of two cantilever trusses of the timber framework that had supported a canvas roof of the Cavalcade (see Fig. 17). The structure had been in service,

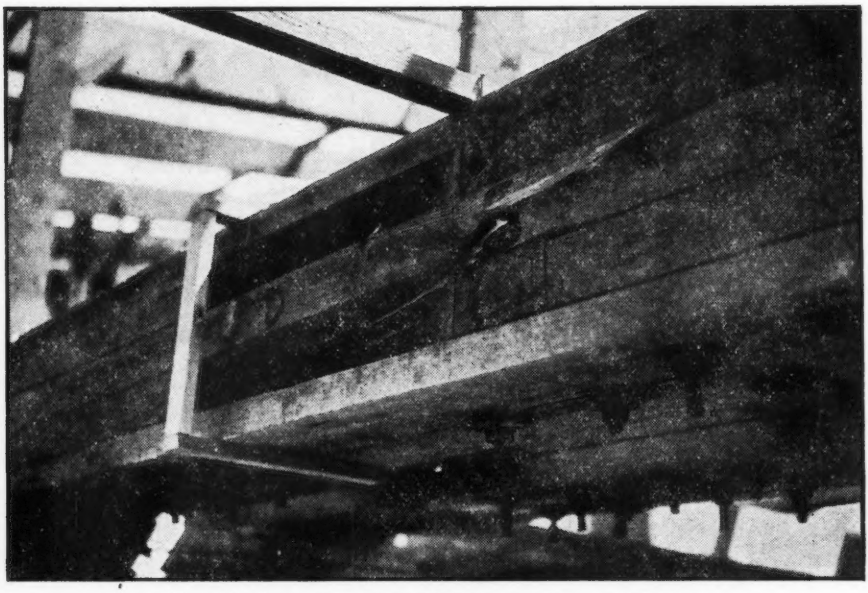


FIG. 14.—FAILURE OF TRUSS TB-2 UNDER A LOAD OF 17.2 KIPS

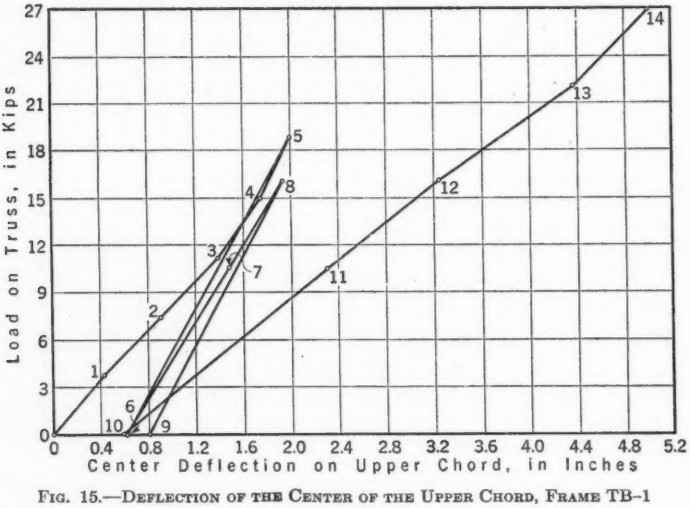


FIG. 15.—DEFLECTION OF THE CENTER OF THE UPPER CHORD, FRAME TB-1

exposed to the elements, for about 2 years prior to testing, and appeared to be in unusually good condition. The framing lumber was Douglas fir, appraised as "select structural" grade.

Important joints in the framing were bolted or joined by connectors, the latter involving flush-type shear plates (Fig. 18) with steel gusset

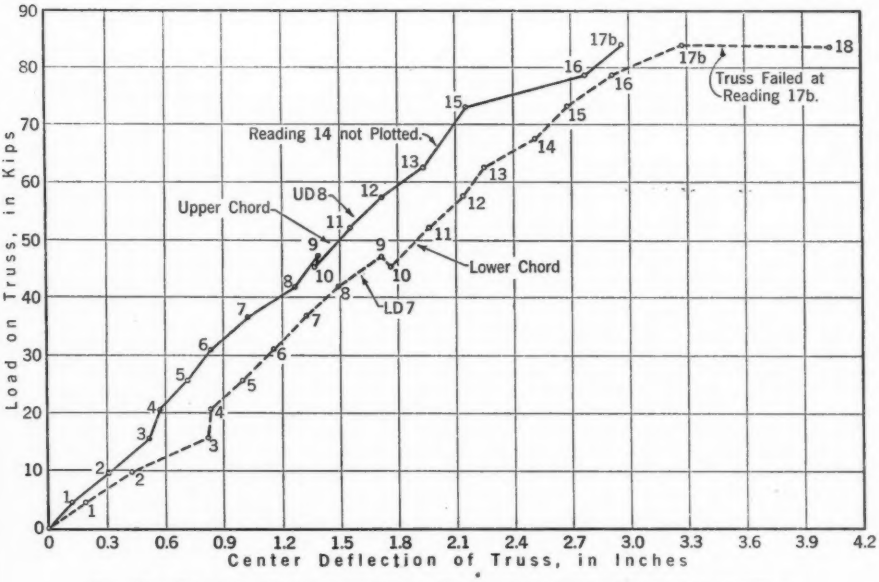


FIG. 16.—RELATIVE MOMEMENTS, THREE PARTS OF THE LOWER CHORD, FRAME TB-1 (FOR LOCATION OF GAGES, SEE FIG. 13)

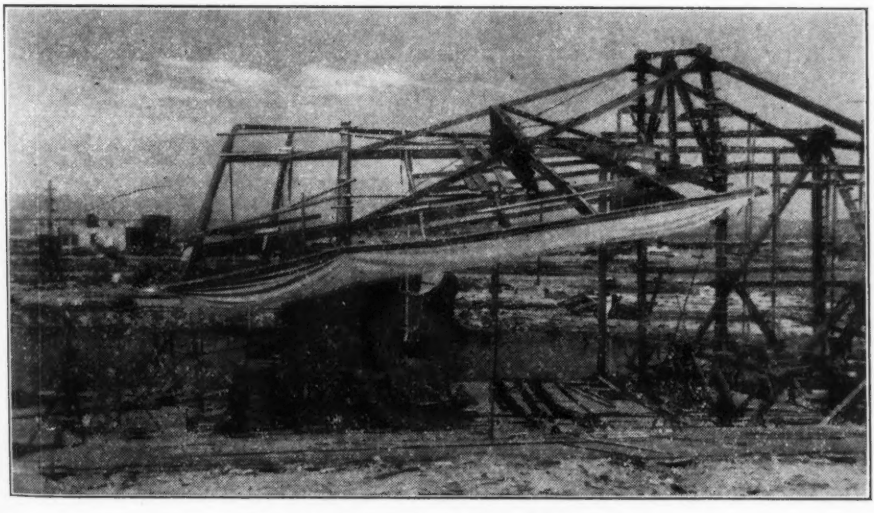


FIG. 17.—FAILURE OF A CANTILEVER FRAME

plates and split rings. Unless otherwise indicated, all bolts in Fig. 18 were $\frac{3}{4}$ in. in diameter. All bolts, including those at split-ring connectors were provided with 3-in. by 3-in. by $\frac{5}{16}$-in. washers. No bolts were tightened or

frames repaired prior to the application of test load. The gusset plate at joint 6S, Fig. 18, shows the typical minimum dimensions for such plates.

Testing.—Loads were applied by filling water tanks suspended by rope slings. These were fastened about 33 ft from the base of the cantilever. The calibrated tanks were set on temporary jack supports to permit controlled application of load. A preliminary load was applied to one of the pair of

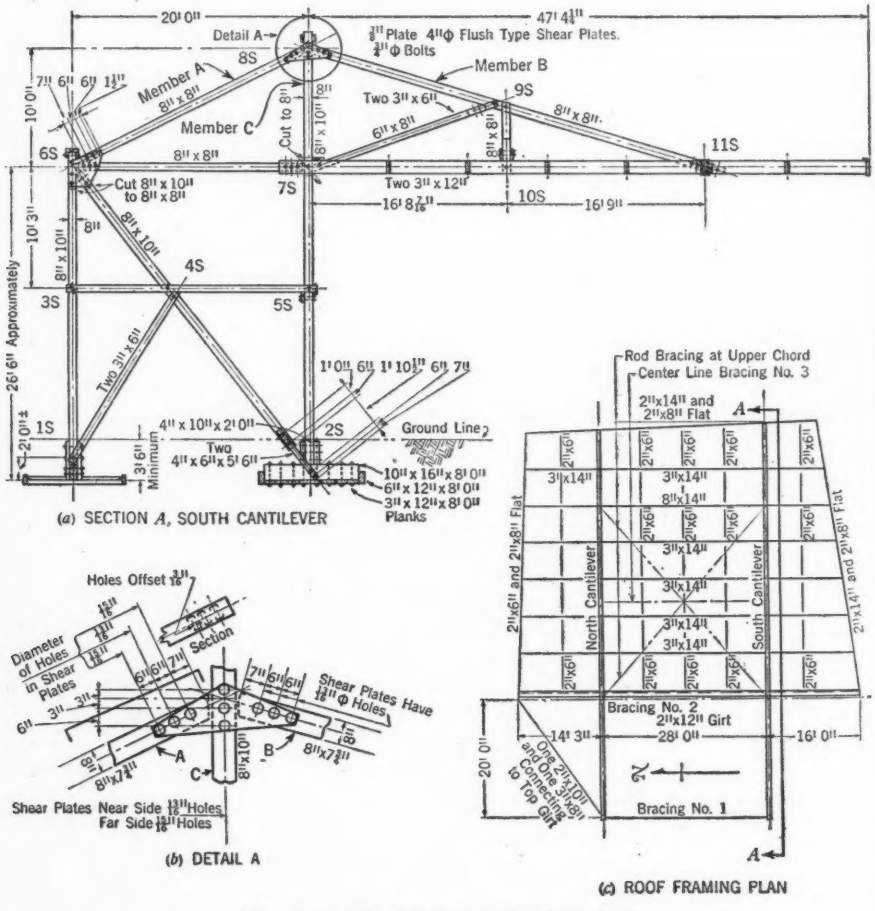


FIG. 18.—DETAILS OF CANTILEVER FRAME TC–1

cantilever frames, to check the workability and safety of the loading apparatus, by slowly releasing the supporting jacks. Shortly after load transfer occurred, the anchor arm of the crown joint (8S in Fig. 18) failed by the shearing out of shear plate connectors and bolts, as shown in Fig. 19.

The calculated load in the failed member was 64,800 lb although, as shown on later examination of the joint, a lesser load may have caused the progressive failure that occurred. The deflection observed at the end of cantilever at time of failure was about 8 in.

Results of Tests, Trusses TC.–Joint 8S: (Fig. 18) in which failure occurred consisted of three 8-in. by 8-in. timber members connected by $\frac{1}{4}$-in. steel plates drilled with $\frac{13}{16}$-in. diameter holes and three pairs of shear plates, 4 in. in diameter, fitted with $\frac{3}{4}$-in. bolts.

Although the character of the failure was typical—that is, a shearing out of wood and pulling through of bolts causing splitting of the members—failure occurred at less than the anticipated total load. An examination of the joint after test indicated that the center pair of three pairs of shear plates, 4 in. in diameter, had holes $\frac{13}{16}$ in. in diameter. The other two pairs had $\frac{15}{16}$-in. diameter

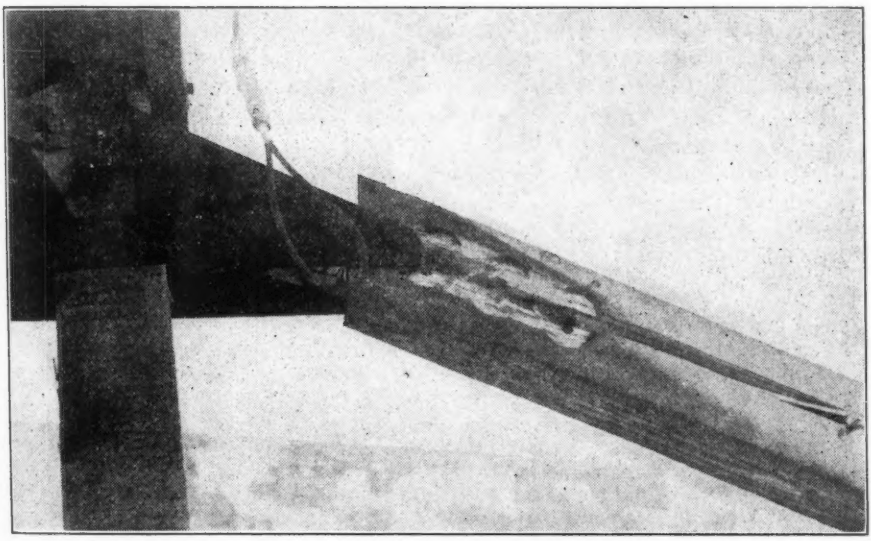


FIG. 19.—FAILURE BY SHEARING OF PLATE CONNECTORS AND BOLTS

holes, although all were fitted with $\frac{3}{4}$-in. bolts. The bolt holes in member A (which were bored halfway through the timber from each side) were found to be offset between opposite faces of the timber about $\frac{3}{16}$ in. Excessive loading of connectors and bolts was evident in member A by the $\frac{1}{16}$-in. indentations in the bolts at the contact points between shear plates and steel gusset plates, and by the distortion of bolt holes in connectors and gusset plates, which indicated that the ultimate shearing capacity of each bolt had been approached. These observations indicated that the connectors were loaded progressively, probably not more than two connectors being loaded at one time.

Although the fabrication of members B and C (Fig. 18) was found to be essentially correct, the three shear plates on one side of compression member C were found to have $\frac{15}{16}$-in. diameter holes instead of the required $\frac{13}{16}$-in. diameter holes for $\frac{3}{4}$-in. bolts. No evidence of distress was found in tension member B or in compression member C.

On the basis of the foregoing estimated failure load, the computed tension in member A was 64,800 lb, or a load of 10,800 lb per shear plate; and an ap-

proximate deflection at the end of the cantilever of about 8 in. was observed at time of failure. The possible influence of secondary stresses due to cantilever deflection, using the foregoing load, was investigated. The computed deflections were:

Cause	Deflection (in.)
Elongation of members	3.17
Slip of joint fasteners	3.31
Total	6.48

Further computations indicated that, with such total calculated deflection, a load of 700 lb per shear plate was induced in member A acting perpendicular to the grain, with an average load of 10,800 lb per shear plate due to direct stress simultaneously acting parallel to the grain. Thus, it appeared that failure in member A, in only a slight degree, was related to the action of secondary stress induced by such deflection.

If the test load at failure was the ultimate load of the truss, the value of R_{ca} for six shear plates is equal to 2.81, although the faulty construction probably caused only two shear plates to act effectively at any one time. Failure of this cantilever frame was obviously the result of careless workmanship. (In the manuscript report on joint tests, R_{ca} for joint J11 is given as 3.12.)

Examination of the joint after the test showed entirely normal performance of individual connectors, such as would be expected when connectors are loaded to failure. Splitting, from wedge action of bolts, could be taken as the effect of secondary stresses, but it has been shown that these were of slight importance.

Although the sudden failure of the truss was regrettable, the test of this existing full-scale structure has genuine value in emphasizing the essential necessity of competent engineering supervision of structures if the design intentions are to be executed satisfactorily; and economic and adequately safe structures are to be provided. It is important to observe that the truss, improperly constructed as it was, had a capacity of nearly three times its working load, and was therefore a safe structure.

Chinese Building (Ch) *Trusses.*—Tests on the Chinese Building were of five similar all-timber trusses with bolted joints. Each truss had a span of 36 ft 8 in.; and, after removal from the building, these trusses were stored outside for about 4 months before testing. When tested, their moisture content was about 12%.

Fig. 20 shows a detail of one of the trusses. All truss members were single sticks, of 2-in. dimension stock, No. 1 common Douglas fir. The compression diagonals were framed on one side of the chords and the tension verticals on the opposite side. This detail was later found to be an important factor in the behavior of the trusses under test loads. Single bolts, with cut-steel washers under heads and nuts, were used in all joints except the four end joints. Bolts were found to be reasonably tight at the time of testing; they were not tightened. Although there were a number of knots, none seemingly reduced

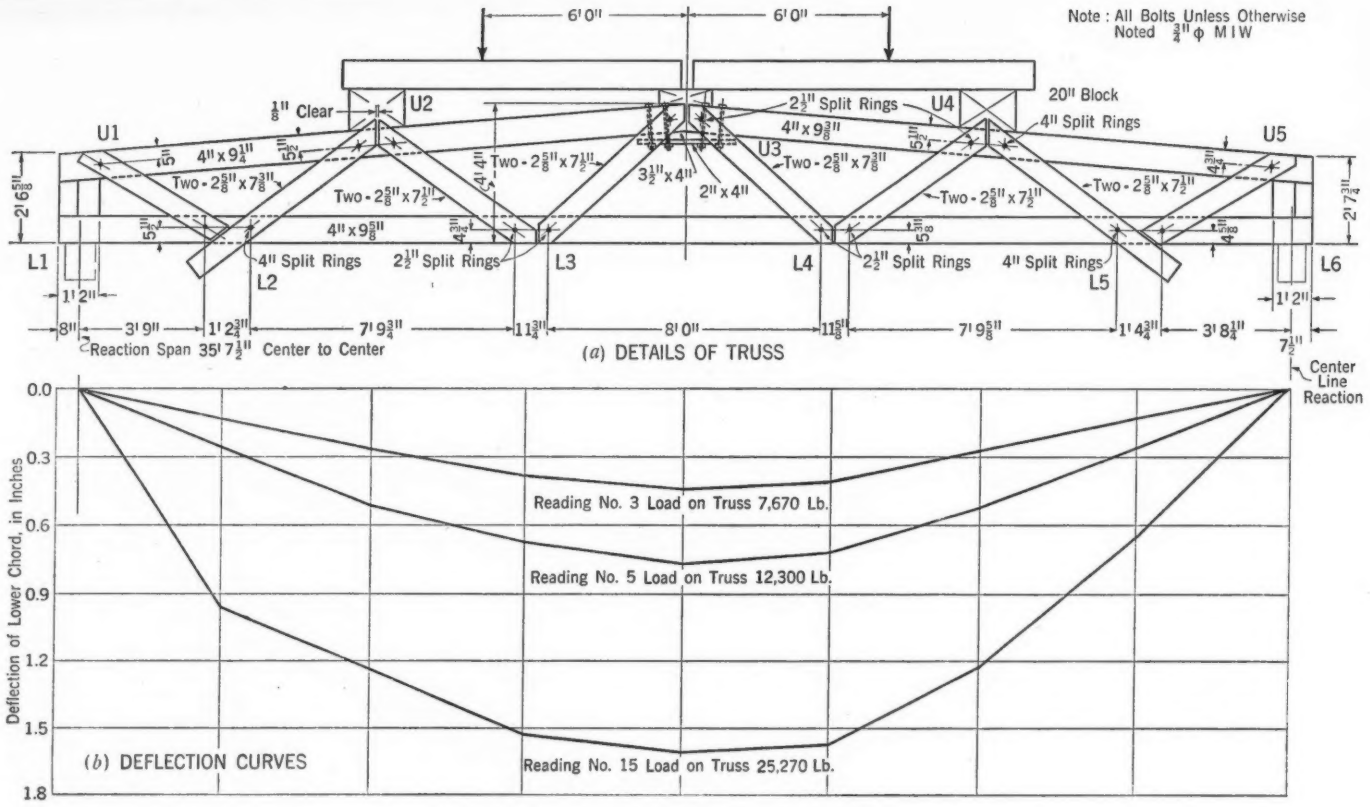


FIG. 20.—TYPICAL TRUSS, THE CHINESE BUILDING

the strength of the truss. Members were relatively free of checks except verticals U1–L2 and U5–L6, in which 1⅛-in. bolts had end distances of only 3¾ in.—and checking had started in several instances.

Testing.—The trusses were tested with concentrated loads while supported by hangers from the laboratory roof, as shown subsequently in Table 9. In the building from which the trusses were taken, the roof sheathing had been nailed directly to the top chords, thus giving them lateral support. In the tests short pieces of 2-in. by 8-in. timber were nailed to these chords to form T-sections between hangers, thus providing equivalent lateral support. As usual, deflection readings, as well as deformation measurements, were made at various points.

The peculiar framing of compression diagonals and tension verticals, previously noted, resulted in a twisting of the chords, and a bowing of the web members, under test loads, as shown in Fig. 21*A*. The amounts were measured by gages. It is judged that the method of supporting the chords during testing gave about the same resistance to this twisting and bowing as in the original structure.

Trusses TCh–1 and TCh–3 failed in the lower chord splices, each of which had four ⅝-in. bolts and a few nails. Consequently, the remaining trusses, TCh–2, TCh–4, and TCh–5 were reinforced at this splice, so that failure would occur elsewhere. Truss TCh–2 failed by combined bending and direct stress in the tension vertical U1–L2, as shown in Fig. 21*B*.

Trusses TCh–4 and TCh–5 failed by the shearing out of the wood in the end distances in verticals U1–L2, and by the splitting of the chords. In truss TCh–5, the nut had been omitted from the 1-in. bolt connecting the three members at joint U1. This truss proved to be the strongest of all five, possibly because of the lack of a nut resulted in a reduction of bending moment in member U1–L2, which, with the exception of the lower chord splices, was the weakest factor in all trusses. The omission of the nut threw a larger part of the moment to the end diagonal, and this member, a 2-in. by 10-in. stick, was amply strong.

Results of Tests, Trusses TCh.—The results of the tests are given in Table 3. Fig. 20 shows typical deformation curves for the lower chord for three separate systems of loads on the truss. Fig. 22 shows a typical center deflection curve of the lower chord as plotted against the total truss load. Fig. 23 shows another load deflection curve, for the condition in which one of the trusses was loaded to about 90% of its ultimate capacity (reading No. 11); the load was then released, measurements taken, and the load reapplied. In the test, a permanent set of 1/330 of the span was noted, when the loads were released.

The failure of truss TCh–2 merits explanation. The eccentric connections which twisted the chords subjected the web members to bending moments resisting this twist. In the vertical U1–L2, the calculated stress in the 2-in. by 4-in. vertical, at the bolt where failure occurred, was 1,030 lb per sq in. in tension, and 8,400 lb per sq in. in bending—a total of 9,430 lb per sq in., representing an R_{wa}-value of 5.9 based on 1,600 lb per sq in. of allowable combined stress. The amount of twist in the chords is shown graphically in

Fig. 24. As mentioned previously, this vertical U1–L2 had very short end distances for the size of its connection. In all the trusses, splits occurred in

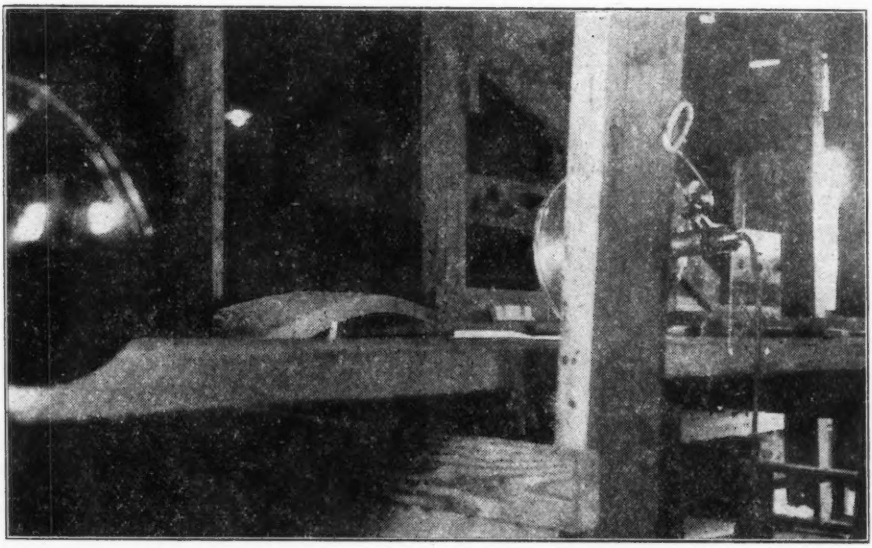


FIG. 21*A*.—TWISTED CHORD AND BOWED WEB MEMBER, UNDER TEST LOADS

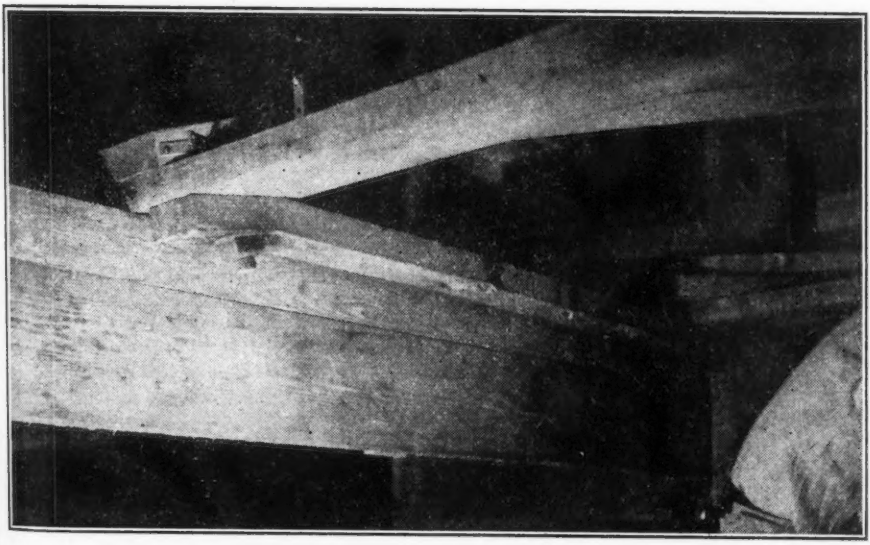


FIG. 21*B*.—TENSION AND BUCKLING FAILURE OF A VERTICAL MEMBER

these end distances at loads below the ultimate capacity of the truss; nevertheless, the splits did not directly cause failure of the trusses. If the allowable load in the verticals is taken as the sum of allowable loads on the bolts (2,200

TABLE 3.—SUMMARY OF TEST RESULTS; STRESSES IN THE TRUSSES OF THE CHINESE BUILDING (Ch)

Truss	Center Deflections (Fractions of Span)			Location of truss failure	Total Maximum Load on Truss		
	At P = 1,850 lb on truss	At Other Loads			(Pounds)	(Lb per lin ft of truss	(Lb per sq ft of roof)
		P (lb)	Deflections				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
TCh–1...	1/690	7,390	1/323	Lower chord splice	8,980	350	175
TCh–2...	1/870[a]	10,160	1/141	U1–L2[b]	11,380	443	222
TCh–3...	1/685[a]	9,070	1/119	Lower chord splice	9,970	387	194
TCh–4...	1/783	7,450	1/169	Joint U1 in U1–L2[c]	9,420	367	184
TCh–5...	1/885	8,910	1/149	Joint U1 in U1–L2[c]	11,680	455	228

[a] These are approximately the deflections caused by design loads, which correspond with 72 lb per lin ft of truss, or 36 lb per sq ft of roof. [b] Broke in combined tension and bending at point L2. [c] Shear in the end distance of the 1-in. bolts at joint U1 in member U1–L2; then the chords split from the twisting action.

TABLE 3.—(*Continued*)

Truss	Total Maximum Stress (Lb)		R_{ls} of splice[f]	R_{vs} at U1–L2[h]	Ratios R_{ls} Based on Bolt Values[j]			Total Elapsed Time	
	On U1–L2 at failure[d]	Lower chord splice			At Point of Failure;[k] with End Distance		Design values	Hr	Min
					Neglected	Included			
	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
TCh–1..	3,628	14,319	5.3	....	....	....	....	4	20
TCh–2..	4,594	(Reinforced)	....[g]	5.9[i]	2.07	4.14	6.4	2	30
TCh–3..	4,025	15,896	5.7	....	....	....	....	3	0
TCh–4..	3,803	(Reinforced)	....[g]	....	2.0	3.0	5.9	1	50
TCh–5..	4,791[e]	(Reinforced)	....[g]	....	2.4	4.9	7.3	2	15

[d] Ends of members U1–L2 and U5–L6, that were not checked, split at stresses, in the members, equal to from F = 2,100 lb to F = 3,600 lb. [e] Nut omitted at U1. [f] Using a value of 90 lb for each assisting 16-penny nail. [g] Did not fail in splice because it was reinforced. [h] At the point where member U1–L2 broke in bending. [i] Based on an allowable load of 1,600 lb per sq in. in bending. [j] Allowable loads on member U1–L2 as governed by bolts, based on their values in the 2-in. by 4-in. stick, parallel to the grain, and without reducing for the short end distance, is 2,200 lb at joint L2 and 1,960 lb at joint U1. To allow for short end distances these values are reduced 50% to 1,100 lb and 980 lb, respectively. Allowable loads on U1–L2 as governed by the bolts, based on their values in the chord, across the grain, are 715 lb at joint L2 and 653 lb at joint U1. [k] In member U1–L2 not in chord.

lb at L2 and 1,960 lb at U1) as given in footnote[j], Table 3, it is seen that these splits occurred early in the progress of the tests. They evidence the effect of short end distances for bolts.

No tests appear to have been made on the complicated action of the bolts as found in these joints; hence, the question arises as to the allowable bolt values for such details. In Table 3, the ratios for the joints L2 and U1 (see Col. 14, Table 3) are based on the perpendicular to the grain value of bolts as given in Col. 9. Table 3 neglects the action of the diagonal on the far side. From this point of view, the ratios would seem ample. On the other hand, the failures in trusses TCh–2, TCh–4, and TCh–5 were in this vertical itself

(U1–L2) where the controlling value would be the parallel to grain value of the bolts, without reducing for short end distance (see footnote[f], Table 3).

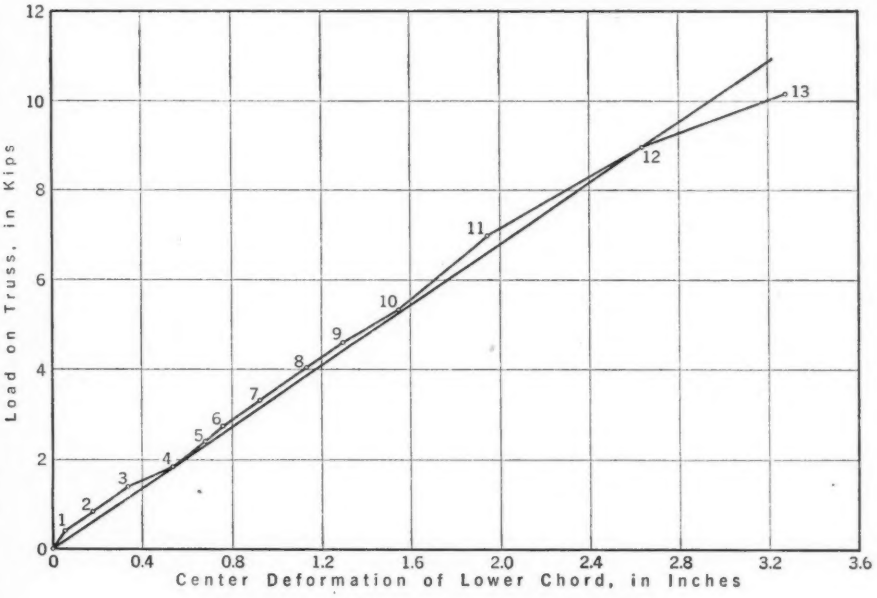


FIG. 22.—LOAD-DEFLECTION CURVE FOR TRUSS TCH

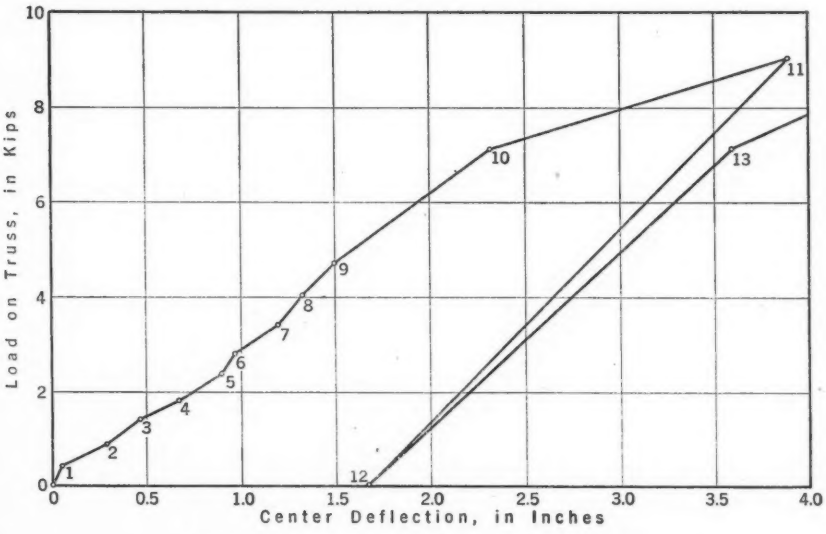


FIG. 23.—LOAD-DEFLECTION CURVE FOR TRUSS TCH-3

Based on these values (a procedure which seems more nearly correct), the ratios R are lower.

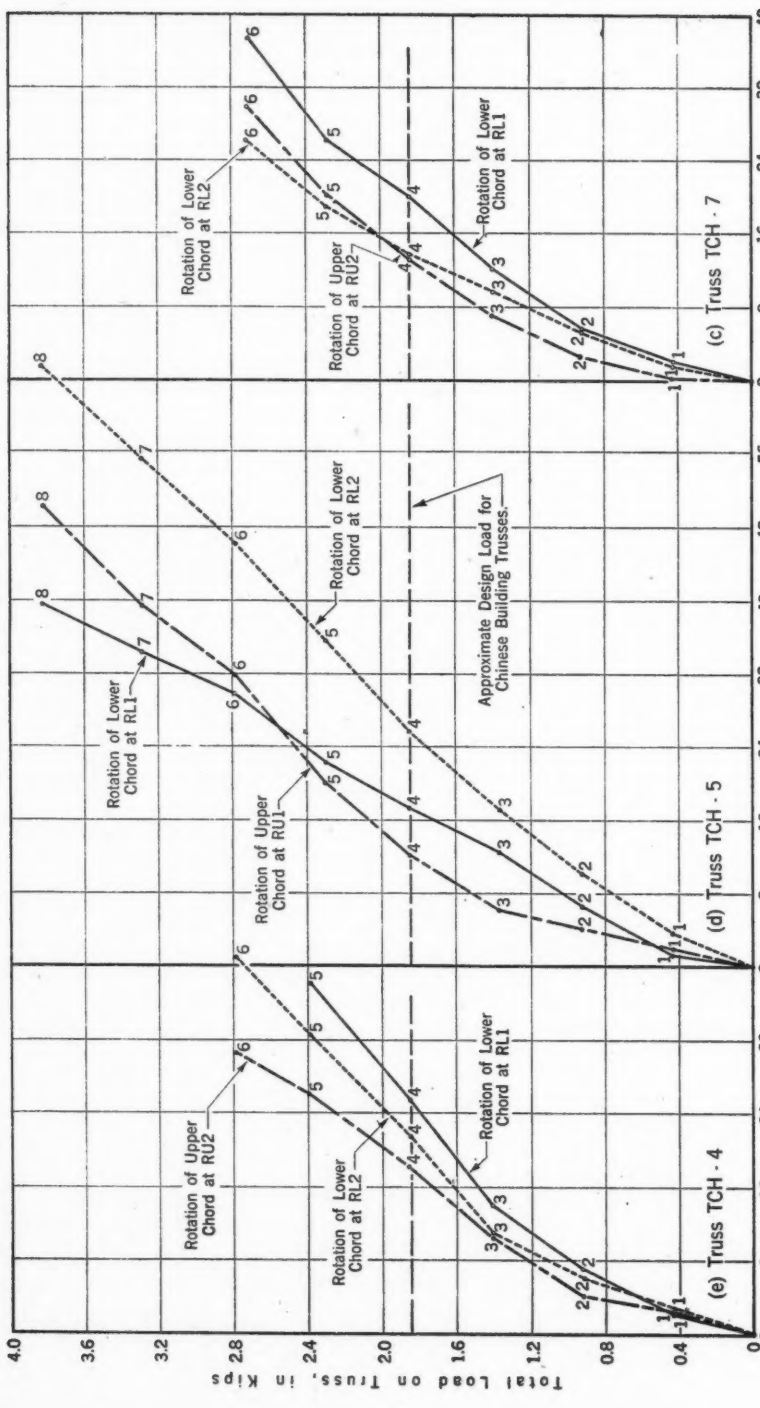

Total Load on Truss, in Kips
Rotation of Chords, in Minutes of Angle
(e) Truss TCH · 4
(d) Truss TCH · 5
(c) Truss TCH · 7
Rotation of Upper Chord at RU2
Rotation of Lower Chord at RL2
Rotation of Lower Chord at RL1
Rotation of Upper Chord at RU1
Approximate Design Load for Chinese Building Trusses.

FIG. 24.—MEASURED ROTATION OF THE CHORDS OF TRUSSES TCH

In trusses TCh–4 and TCh–5, the chords developed long splits at failure—probably another result of the twisting of the bolts. These trusses were adequate for the functions they served, although the manner of failure was unexpected and not in accordance with results that might have been expected from usual or conventional investigatory computations. A change in the proportions of members would have affected the distribution and magnitude of stresses. Increasing the thickness of chords, for example, would have increased the bending moments in verticals and diagonals and would also have affected the connections. An analysis of additional stresses in truss members and their connections caused by sizes and arrangement of members would seem to be justified when such factors influence the carrying capacity of the truss.

Apparently, the use of cut washers did not decrease the strength of the trusses. When bolts are subjected to shear only, or when the washers become loose through shrinkage of the wood, the size of the washers appears to have little influence upon the strength of the connection.

The strength values of bolts subjected to twisting, or bolts passing through members subject to twisting should be investigated further; likewise, any tendencies of the bolts to split the members they connect should be determined. This subject is probably related to the strength of bolts passing through loose fillers—a subject on which authoritative data are much needed. The tests indicate the importance of proper end distances; they are important factors in the strength of the connections of a truss.

French Building (F) *Truss.*—This truss, spanning 37 ft, was built of wooden members connected eccentrically at the joints with split rings and bolts as shown in Fig. 25. All web members were diagonals, of two $2\frac{5}{8}$-in. by $7\frac{1}{2}$-in. sticks each, and each diagonal was connected at each end to the single stick, 4-in. by 10-in. chords with one pair of split rings with $\frac{3}{4}$-in. bolts.

While dismantling the building, web members U1–L2 and L5–U5 were damaged, and these members were replaced. Otherwise, no repairs were made. Careful examination of the truss disclosed no other damage than that noted. Wood was of dense select structural grade, Douglas fir, and had a moisture content of 12% at the time of testing.

The truss was tested in the usual manner, loads being applied as shown diagrammatically in Fig. 25, producing approximately equal loads at the panel points. Truss deflections of the chords and joint deformations were measured in the usual manner.

After 1 hour and 13 min of testing, with a total load of 15.00 kips on the truss, the joint at L5 cracked audibly and the load dropped to about 5.0 kips. It was observed that cracks about $\frac{1}{4}$ in. wide had developed on the wide faces of the chord, these cracks appearing to be similar to the failures of the North Square Court Truss (TNS), described as tension perpendicular to the grain. Loads were then almost completely released, and four $\frac{1}{2}$-in. stitch bolts with large plate washers were installed in the chord at joint L5, and tightened. Similarly, four stitch bolts were installed at joint L2, but these were left untightened so that they would not interfere with further testing. Loads were then reapplied; and, 1 hour and 14 min later, a large split developed at

FIG. 24.—MEASURED ROTATION OF THE CHORDS OF TRUSSES TCh

FIG. 25.—TYPICAL TRUSS, THE FRENCH BUILDING

joint L2 when the truss was loaded with 27.1 kips. This failure, shown in Fig. 26, was apparently also caused from tension perpendicular to the grain. The photograph reveals that the split is widest at the center line of the joint, tapering off to nothing farther from the joint. After this failure the loads were released and the stitch bolts at joint L2 were tightened. Loads were

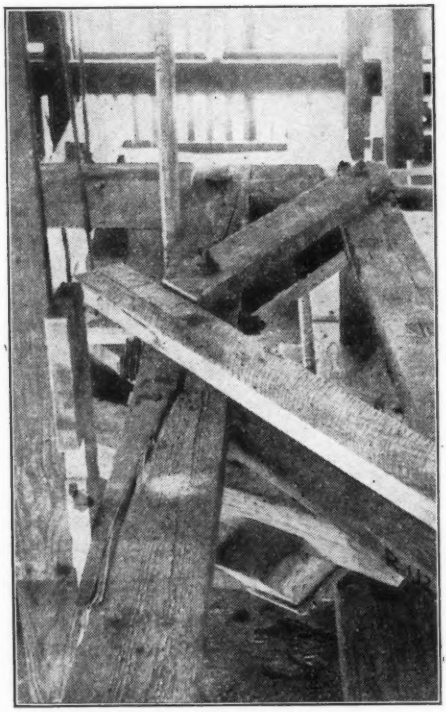


FIG. 26.—FAILURE OF TRUSS TF AT JOINT L2, BEFORE TIGHTENING THE STITCH BOLTS

reapplied, when, at a maximum load of 31.2 kips, joint L2 failed again along the original split. In this second failure, because the chord member was held tight together at the joint by stitch bolts, the split grew wider as it went farther from the joint. The widening of these splits was apparently caused by the large bending stresses induced through the eccentric connection which tended to rotate the chord. Immediately after failure, the truss was sustaining a load of 21.7 kips, which was allowed to remain overnight. The following morning this load had dropped to 18.5 kips.

Results of Tests, Trusses TF.—The results of these tests can be summarized as follows:

Item	Description	Quantity
	Deflection of Truss at an Allowable Load of 9.8 Kips (Total Load Based on Dense Select Structural Grade):	
1	In inches	0.60

2	Percentage of span	1/720
	Maximum Recorded Deflection (Truss Load, 25.3 Kips):	
3	In inches	1.62
4	Percentage of span	1/270
	Maximum Truss Load, in Kips:	
5	At first failure (joint L5)	15.0
6	After reinforcing	31.2
	Ratio R_{ca} as Limited by the Strength of:	
7	Joint L2	3.2
8	Joint L5 unreinforced	1.5
9	Joint L5 reinforced (did not fail and showed no sign of distress)	3.1
	Average Unit Shear, in Pounds per Square Inch, at Failure in:	
10	Joint L2	450
11	Joint L5 unreinforced	214
12	Joint L5 reinforced (did not fail)	450

Stresses in the members (Table 4) were computed by an approximate method that takes the eccentricities in the joints into account. Deflection curves for the bottom chord of the truss for three loadings are shown in Fig. 25. Fig. 27 indicates the center deflection of the truss. The estimated allowable load W_{ca} on the truss was 9.8 kips, based on the connections of the members that failed.

TABLE 4.—STRESSES FOR AXIAL LOADS IN MEMBERS AT FAILURE (KIPS)

Description	U1–L2	U5–L5		U2–L2	U4–L5	
		Unreinforced	Reinforced[a]		Unreinforced	Reinforced[a]
Stress	36.7	17.1	36.7	28.7	15.0	31.3
Vertical component	18.5	9.0	18.8	17.7	9.4	19.6
Stress, s_{ca}	11.5	11.5	....	....	....	....

[a] The reinforced joints did not fail.

Prior to reinforcing the chords with stitch bolts, the ultimate loads were about one and one-half times the estimated allowable load, giving a ratio R_{ca} of 1.7. After the stitch bolts had been provided, the capacity of the truss was approximately doubled, with $R_{ca} = 3.1$, indicating that the stitch bolts would be of considerable value if they could be kept tight. The nature of the failures of this truss emphasizes again that the stress of tension perpendicular to the grain is likely to be the determining factor in the strength of joints in which the members are eccentrically connected. If such connections are to be used, the probable amount of this stress should be determined so that the limiting capacity of the connections will be known.

Merry-Go-Round Building (M) *Trusses.*—Two all-timber trusses (TM–1 and TM–2) of similar design, spanning 54 ft 1 in. and 51 ft 4½ in., and using the double Warren type of framing, were obtained from the Merry-Go-Round Building and tested to failure. Top and bottom chords were approximately parallel, 6 ft 8 in. apart at the center, and all members were connected concentrically using split rings and ¾-in. bolts. Trusses and columns were connected

to form rigid-frame bents for resistance to lateral forces. Details of the trusses are shown in Figs. 28 and 29. In dismantling the building, the trusses were pulled over and fell on the ground, injuring several of their members. Repairs, consisting of the addition of stitch bolts and pads, were made to truss TM–1, as noted in Fig. 28, and to one diagonal and at one reaction point of truss TM–2, as shown in Fig. 29. As a result of the repairing, it is believed that the test results were not affected by the fall of the trusses.

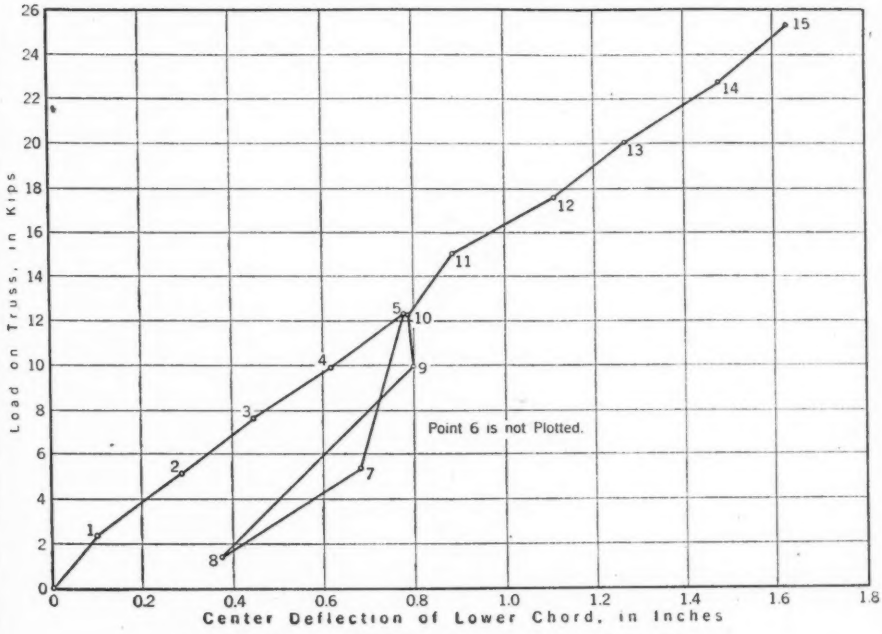


FIG. 27.—CENTER DEFLECTION ON LOWER CHORD, TRUSS TF–3

Timbers were of select structural grade Douglas fir and when tested had a moisture content of about 15%.

Testing.—Loads were applied as shown in Fig. 28. The testing span of TM–1 was 54 ft 1 in., whereas that of truss TM–2, on account of the repairs at the left end, was 51 ft 4½ in. Gages were installed at the top and bottom chords to measure truss deflections, and dial gages were located to measure local joint deformations.

The chords of each half of the trusses were subject to equal and opposite twisting moments. Gages to measure this twisting were installed in locations as shown in Fig. 28.

Truss TM–1 was loaded with 54.2 kips over a period of about 6 hours, when the load was released. Upon reloading to 51.60 kips, failure occurred in the lower chord splice. The splice was then repaired; and, after 21 hours, the truss was reloaded to 72.90 kips, when a check and a bow had developed in the top chord at joint U7. At a load of 81.70 kips a large split appeared in the bottom chord at joint L2 (Fig. 30) and almost simultaneously at joint U2.

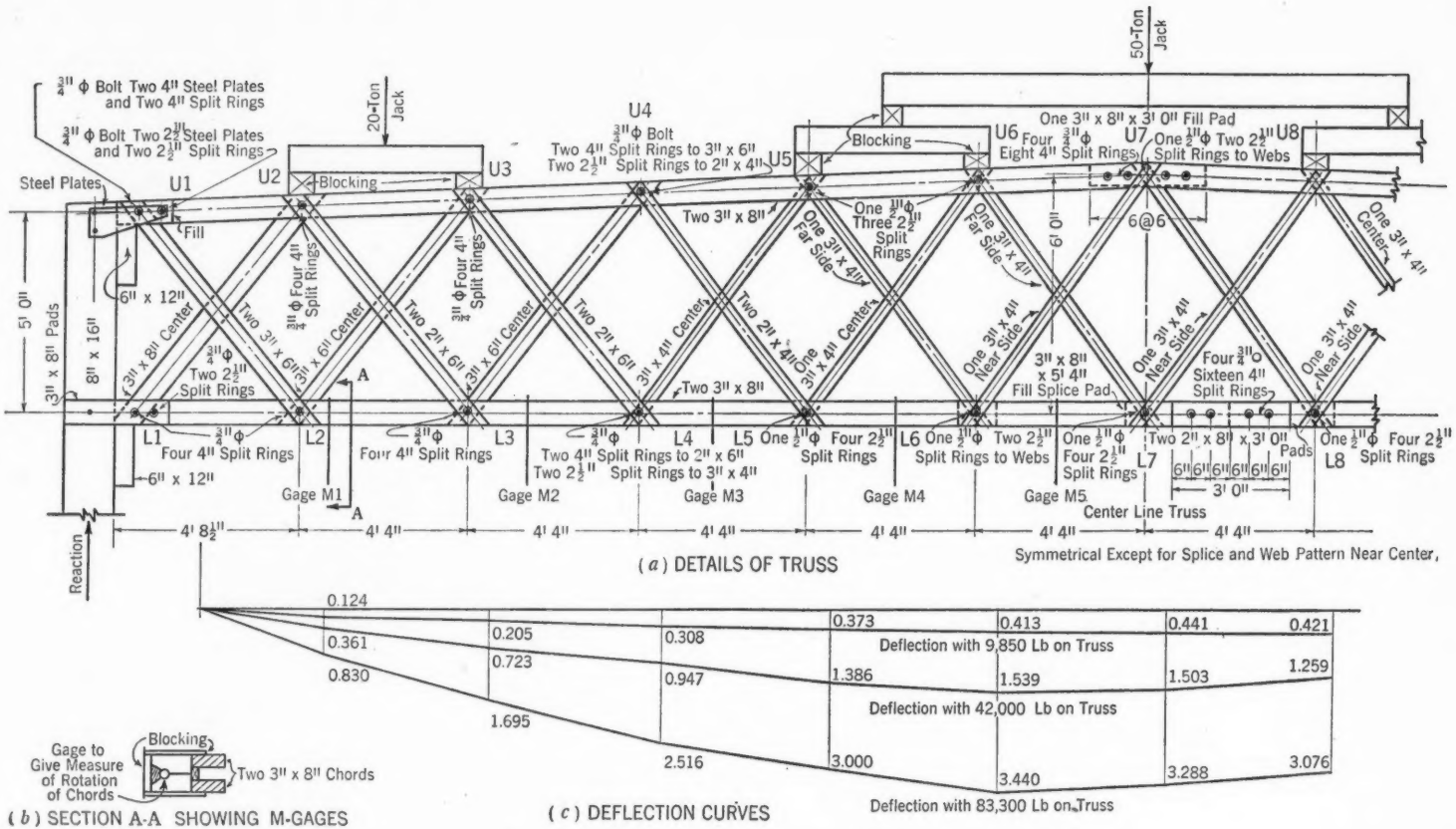


FIG. 28.—TRUSS TM-1, MERRY-GO-ROUND BUILDING

Because of Damage to Left End of TM-2,

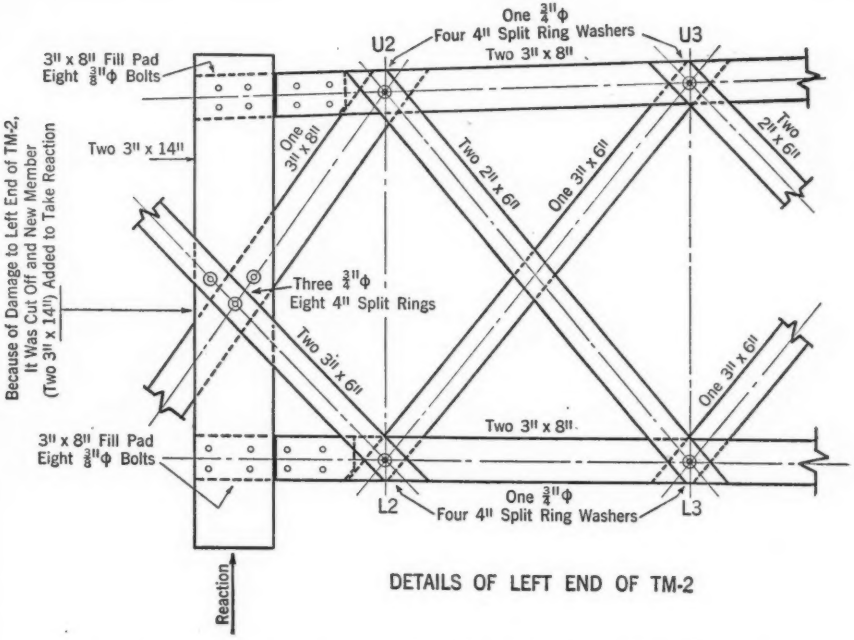

One ¾" ϕ
Four 4" Split Ring Washers
U2
U3
Two 3" x 8"
3" x 8" Fill Pad
Eight ⅜" ϕ Bolts
Two 3" x 14"
Because of Damage to Left End of TM-2, It Was Cut Off and New Member (Two 3" x 14") Added to Take Reaction
One 3" x 6"
Two 2" x 6"
One 3" x 6"
Two 2" x 6"
Three ¾" ϕ
Eight 4" Split Rings
Two 3" x 6"
One 3" x 6"
Two 3" x 8"
3" x 8" Fill Pad
Eight ⅜" ϕ Bolts
L2
L3
One ¾" ϕ
Four 4" Split Ring Washers
Reaction
DETAILS OF LEFT END OF TM-2

FIG. 29.—DETAILS OF LEFT END OF TRUSS TM-2 (REMAINDER OF THIS TRUSS THE SAME AS TM-1, FIG. 28)

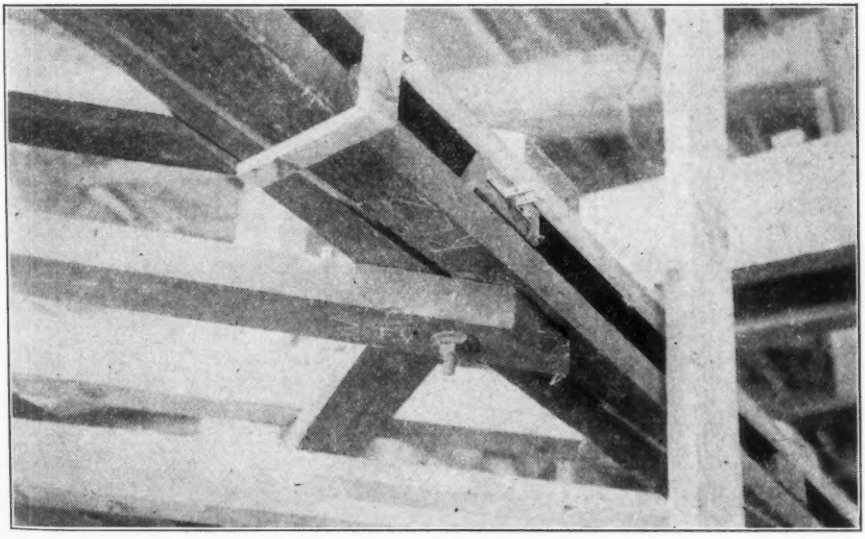


FIG. 30.—FAILURE OF BOTTOM CHORD, TRUSS TM-1, AT JOINT L2

FIG. 28.—TRUSS TM-1, MERRY-GO-ROUND BUILDING

After releasing the load, a total of 83.30 kips was then applied, when these splits became larger and a new failure occurred at joint L2 (see Fig. 31). The diagonal member U5–L4 split and rotated as shown, and failures at other joints then followed in rapid order.

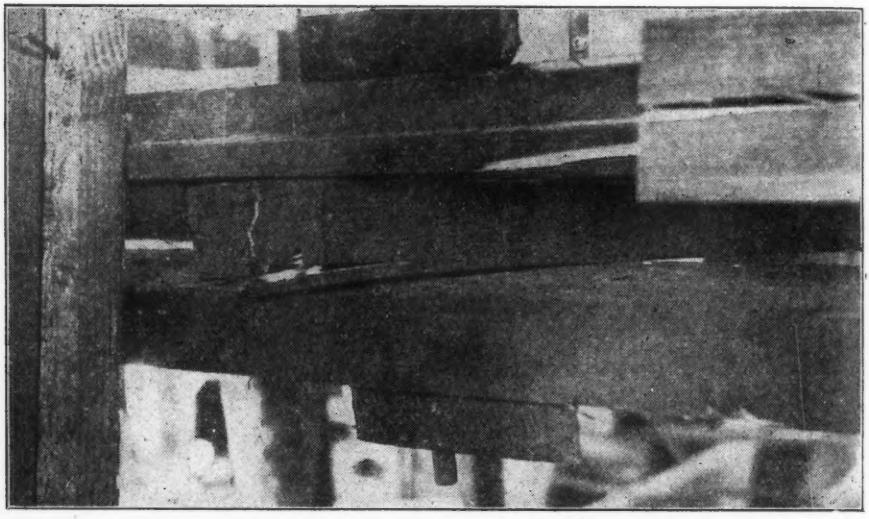


FIG. 31.—ROTATING AND SPLITTING OF DIAGONAL MEMBER AT THE FAILURE OF TRUSS TM-1

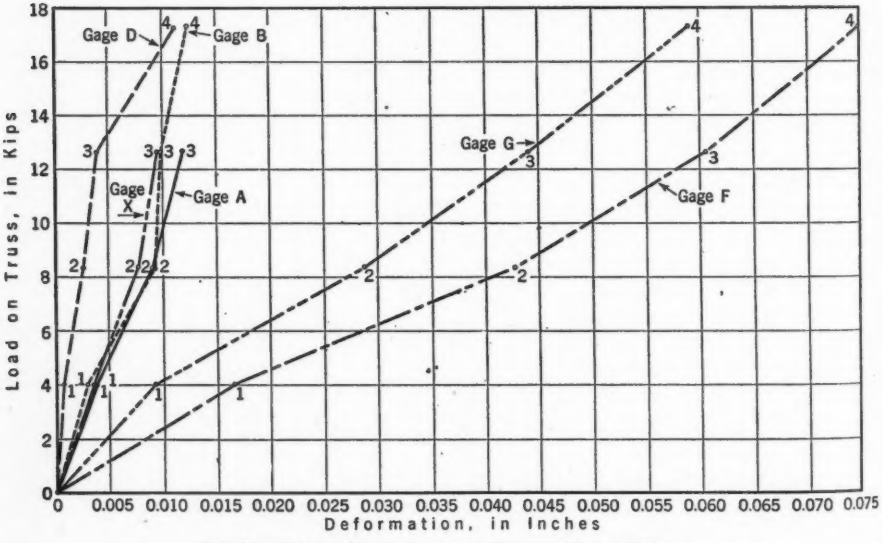


FIG. 32—CENTER DEFLECTION OF CHORDS, TRUSS TM-2.

In view of the experience in testing truss TM–1, the lower chord splice of truss TM–2 was reinforced before testing. When the load had reached 83.30 kips, a crack about 24 in. long opened on one side of the lower chord of joint

L2, and failure occurred at a maximum load of 88.20 kips. The load then dropped to 80.70 kips—which load was allowed to remain on the truss overnight. On the following morning, the truss still carried 62.10 kips.

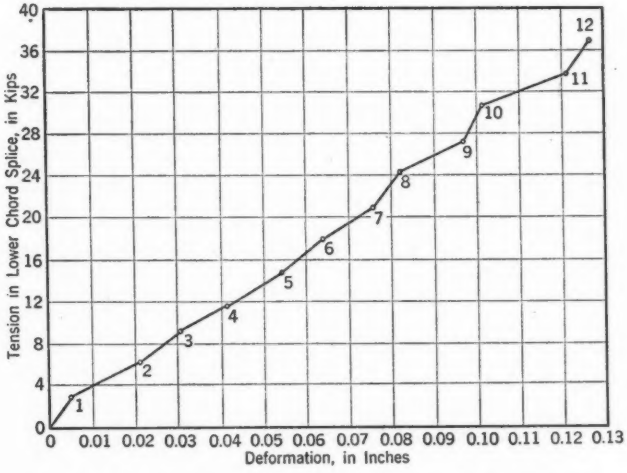


FIG. 33.—DEFORMATION AT THE LOWER CHORD SPLICE, TRUSS TM-1

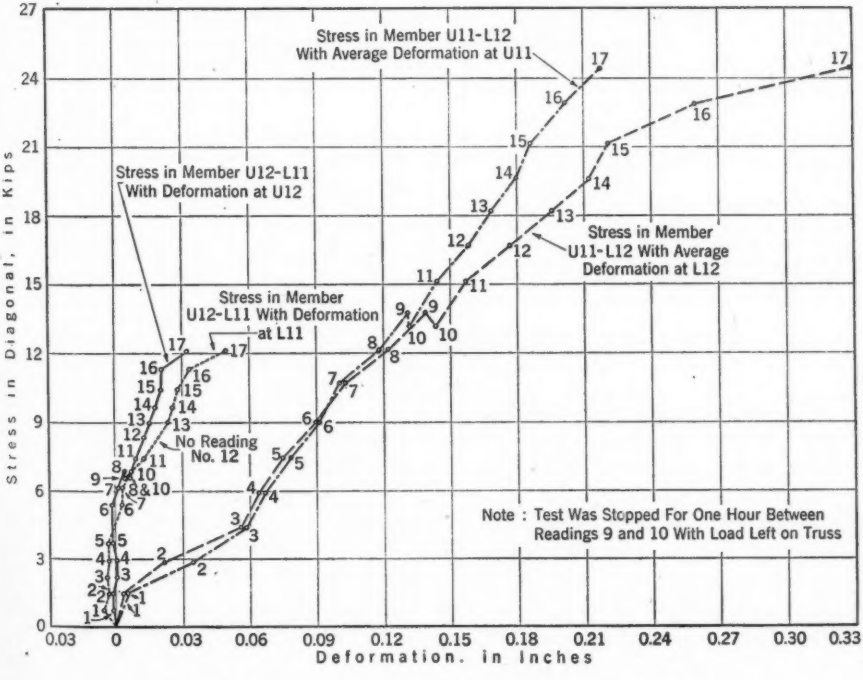

Stress in Member U11-L12
With Average Deformation at U11
Stress in Member U12-L11
With Deformation at U12
Stress in Member
U12-L11 With Deformation
at L11
No Reading
No. 12
Stress in Member
U11-L12 With Average
Deformation at L12
Note : Test Was Stopped For One Hour Between
Readings 9 and 10 With Load Left on Truss
Stress in Diagonal, in Kips
Deformation, in Inches
0.03 0 0.03 0.06 0.09 0.12 0.15 0.18 0.21 0.24 0.27 0.30 0.33
0 3 6 9 12 15 18 21 24 27

FIG. 34.—JOINT DEFORMATIONS, MEMBERS U12-L11 AND U11-L12

Failures in this truss were quite similar to those in truss TM–1—that is, by a lengthwise splitting of the chords. This truss also showed distress simultaneously at various points before the maximum loads were reached. Like truss TM–1, the top chord bowed and split at joint U7. In addition, shear failures in the end distances were noted at several points.

When testing both trusses, cracking noises were heard at loads of from 30 kips to 40 kips which were believed to be caused by shearing of split-ring cores. Upon disassembly of the trusses, some cores of split rings were found sheared. Most of these were near the ends of the trusses in the chord members. At the lower chord splice of truss TM–1, several cores were found broken off with indications that this condition had existed for some time prior to testing; possibly having occurred in wrecking the structure.

Results of Tests, Truss TM.—The significant test data to be recorded for these trusses are:

Item	Description	TM–1	TM–2
	Deflection at a Truss Load of:		
	25.3 Kips (Approximate Design Load)—		
1	In inches	1.1	0.99
2	Ratio of span	1/567	1/630
	81.7 Kips—		
3	In inches	3.6	....
4	Ratio of span	1/173	....
	88.2 Kips—		
5	In inches	....	3.0
6	Ratio of span	....	1/187
7	Maximum load in truss in kips	83.3	88.2
	Allowable Load on Critical Members, in Pounds:		
8	Diagonal U5–L4	4,750	4,750
	All Other End Diagonals, Based on—		
9	49° value in chords	9,200	9,200
10	Reduced end distance in webs	8,600	8,600
11	Chord unit compressive stress, in pounds per square inch, joint U7, at failure	2,530	2,320
12	Ratio of ultimate load to the designer's requirement that roof loads be 30 lb per sq ft for trusses spaced 17.75 ft, center to center	3.3	3.5

In the case of item 9, the chords failed at splice plates, whereas designs based on a reduced end distance in the webs (item 10) did not fail in this manner.

Deflection curves of the bottom chord of the truss for five loadings are shown in Fig. 29. Fig. 32 indicates the center deflections of the top and bottom chord. Fig. 33 shows the total elongation of the lower chord splice under load. Fig. 34 shows deformations at the joint of each end of diagonal members U12–L11 and U11–L12. Fig. 35 indicates the angular twisting of each half of the lower chord at points M1 to M5, as in Fig. 29.

Although the web members are not all of equal section, all have equal connections. The trusses are statically indeterminate but the stresses in the

web members were calculated on the assumption that each web system took 50% of the shears.

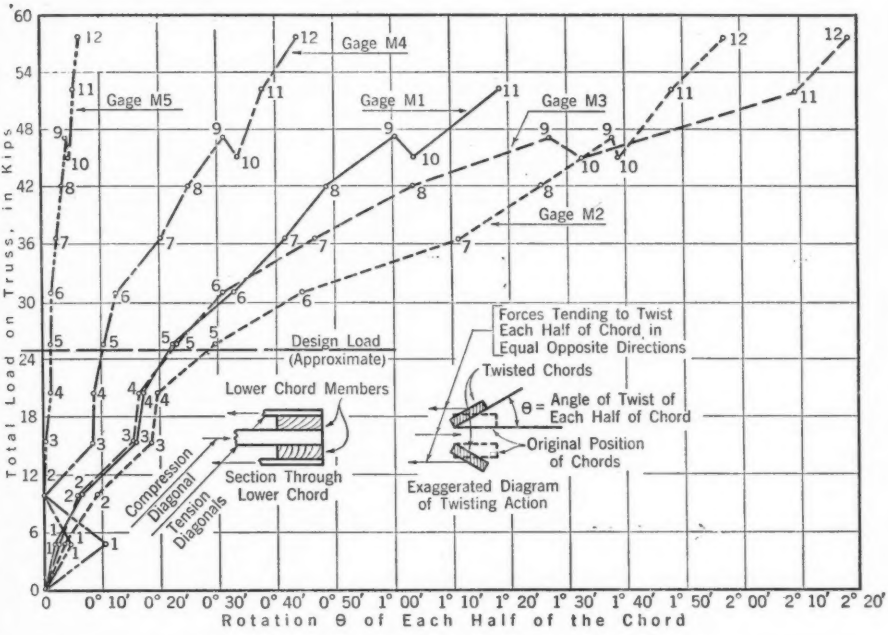

Total Load on Truss, in Kips
Rotation θ of Each Half of the Chord
Gage M4
Gage M5
Gage M1
Gage M3
Gage M2
Design Load (Approximate)
Lower Chord Members
Compression Diagonal
Tension Diagonals
Section Through Lower Chord
Forces Tending to Twist Each Half of Chord in Equal Opposite Directions
Twisted Chords
θ = Angle of Twist of Each Half of Chord
Original Position of Chords
Exaggerated Diagram of Twisting Action

FIG. 35.—ANGULAR TWISTING OF EACH HALF OF THE LOWER CHORD, TRUSS TM-2 (SEE FIG. 29)

In truss TM-1 the first failure occurred in the lower chord splice—possibly the result of damage during wrecking. Therefore, no recommendations or conclusions are presented relative to this failure.

The top chord compression splice at joint U7 showed signs of distress because of uneven bearing on the bevel cuts which caused both splitting and bowing of the members. This bow was accentuated because of shrinkage and also because, in the test, the center panel point was unloaded.

The ratios (R_{ca}) given in Table 5 were based on the working capacity of timber connectors P_c although failure of the trusses came about through twisting and consequent splitting of the chords, and was not due to the connectors.

TABLE 5.—RATIOS R_{ca} BASED ON THE WORKING CAPACITY OF TIMBER CONNECTORS

Member	Stress,[a] F (kips)	Ratio R_{ca}	
		Chords[b]	Web[c]
Truss TM-1:			
U1-L2	27.95	3.05	3.25
U2-L1	28.48	3.09	3.30
U3-L4	12.00	2.53	2.53
U5-L4	12.00	2.53	2.53
Truss TM-2:			
I0-L2	31.00	3.38	3.60

[a] Stresses in diagonals at failure of truss. [b] Ratios based on connector values in chords. [c] Ratios based on connector values in the web. The truss did not fail in this manner because the wood in the end distances did not shear out.

Fig. 36 illustrates one conception of theoretical forces acting on a typical joint. The vertical components of the stresses on diagonal members are

assumed to create bending moments and shears perpendicular to the grain of the chord members. In addition, loads are transferred from rings on one face of the chord to rings on the other face producing "rolling shear" and some tension perpendicular to the grain. (A definition of "rolling shear" is given

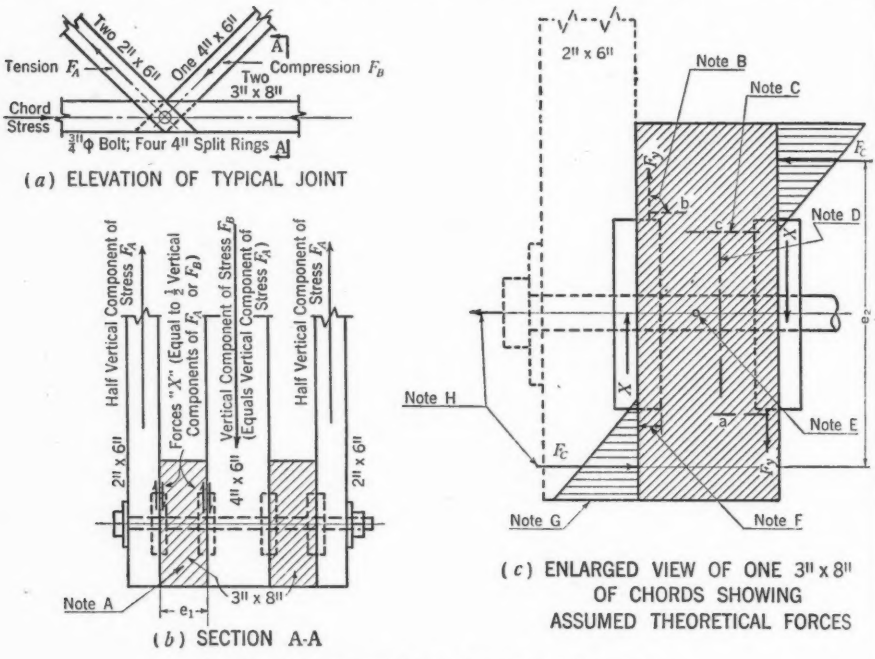


FIG. 36.—THEORETICAL FORCES ACTING ON A TYPICAL JOINT

Note A.—A twisting moment $X\ e_1$ on each 3-in. by 8-in. stick must be resisted, as shown in the enlarged view.
Note B.—Force F_y, the part of the ring load not carried by the core, induces direct tension across the grain on planes a and b and on all points between.
Note C.—Force F_c produces rolling shear on plane c and planes parallel to it.
Note D.—Force X produces rolling shear on this line—any point between wide faces.
Note E.—On this line a bending moment of $\frac{1}{2}F_c\ e_2$ induces tension and compression stresses across the grain.
Note F.—This depth of wood, plus that of the opposite ring (or rings), is not working to resist rolling shear, tension across the grain, direct stresses from F_y, nor bending stresses across the grain from the moment $\frac{1}{2}F_c\ e_2$.
Note G.—Restraining compressive forces F_c induced by the moment $X\ e_1$ (see Note A).
Note H.—Force F_c resisted by bolt tension to the opposite half of the joint.

subsequently under the heading, "Part II. Joint Tests: Compression on Tension Splice Joints with Plywood Splice Plates and Split Rings." The complexity of the problem makes it pertinent to cite a statement by John A. Scholten, as follows:[3]

> "Considerable progress has been made in the theoretical stress analysis of connector joints and in correlating the results with basic data on the mechanical properties of the wood and metal. The fact remains, however, that the stress distribution is so complicated, and the assumptions involved are so often invalid, that actual tests must be relied on to provide the necessary design data."

[3] "Timber-Connector Joints: Their Strength and Design," by John A. Scholten, *Technical Bulletin No. 865*, Forest Products Laboratory, Forest Service, U.S.D.A., Madison, Wis., March, 1944, p. 14.

In thin members, such as these chords, grooves of split rings take out a relatively large area of the chord section, thereby reducing resistance of the chords to stresses caused by the forces shown in Fig. 36.

The tests show that the present methods of design analysis usually employed in trusses of this type (using connector values but neglecting the stress effect on the connected members) may not, and probably will not, reveal the critical stress factors. For example, Fig. 36 illustrates the many forces that may be acting in an apparently simple connection. Because of the wide application of these types of joints, further full-sized tests and study should be continued in an endeavor to establish a rational basis of design.

North Square Court (NS) *Truss.*—Perhaps the most important results obtained in this test program were from the test of a truss taken from the North Square Court area. This truss, with all members of wood, about 10.5 ft deep, spanning about 48 ft, had double stick web members connected eccentrically to the 6-in. by 12-in. chords by 4-in. split rings with $\frac{3}{4}$-in. bolts. Details of the truss are shown in Figs. 37 and 38(*b*).

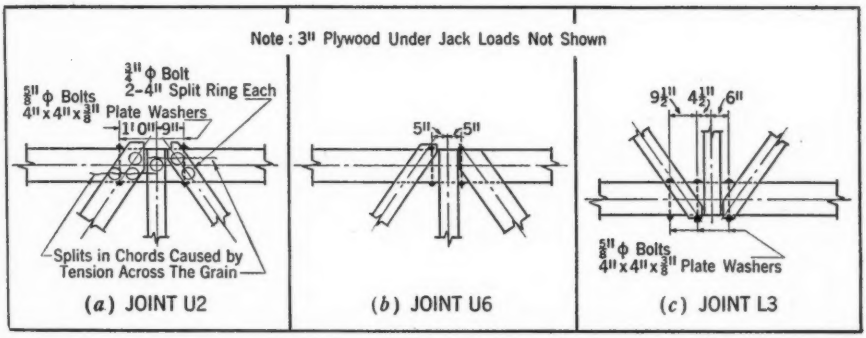


FIG. 37.—TYPICAL JOINTS TNS AND DETAILS OF STITCH BOLTS ADDED DURING TESTS (SEE FIG. 38(*b*))

The workmanship of the truss was excellent, as was evidenced by good fits, indicated by inspection before testing and verified by examination of the disassembled members and their connections after testing. The wood was of structural grade Douglas fir in good condition with the exception of the 6-in. by 12-in. chords. These were of boxed-heart material and had some checks, those in the top chord near joint U6 being about $\frac{1}{4}$ in. wide and extending toward the heart from both faces. For reasons to be explained subsequently, these checks apparently did not affect the test results.

The truss was stored outdoors for about two and one-half months after its removal from the structure and was tested at an average moisture content varying from about 10% to 14%. Because of shrinkage of the timbers, bolts were generally loose from about $\frac{1}{4}$ in. to $\frac{5}{16}$ in.

Testing.—The truss was tested in the typical manner. About one hundred gage points were placed as indicated in Fig. 39 to measure deformations. Loads were applied as shown subsequently in Table 6. Gages LD–1 to LD–11 and UD–1 to UD–13 (see Fig. 39) measured deflections of the truss and indicated bending deformations of the chords caused by eccentric joint connections.

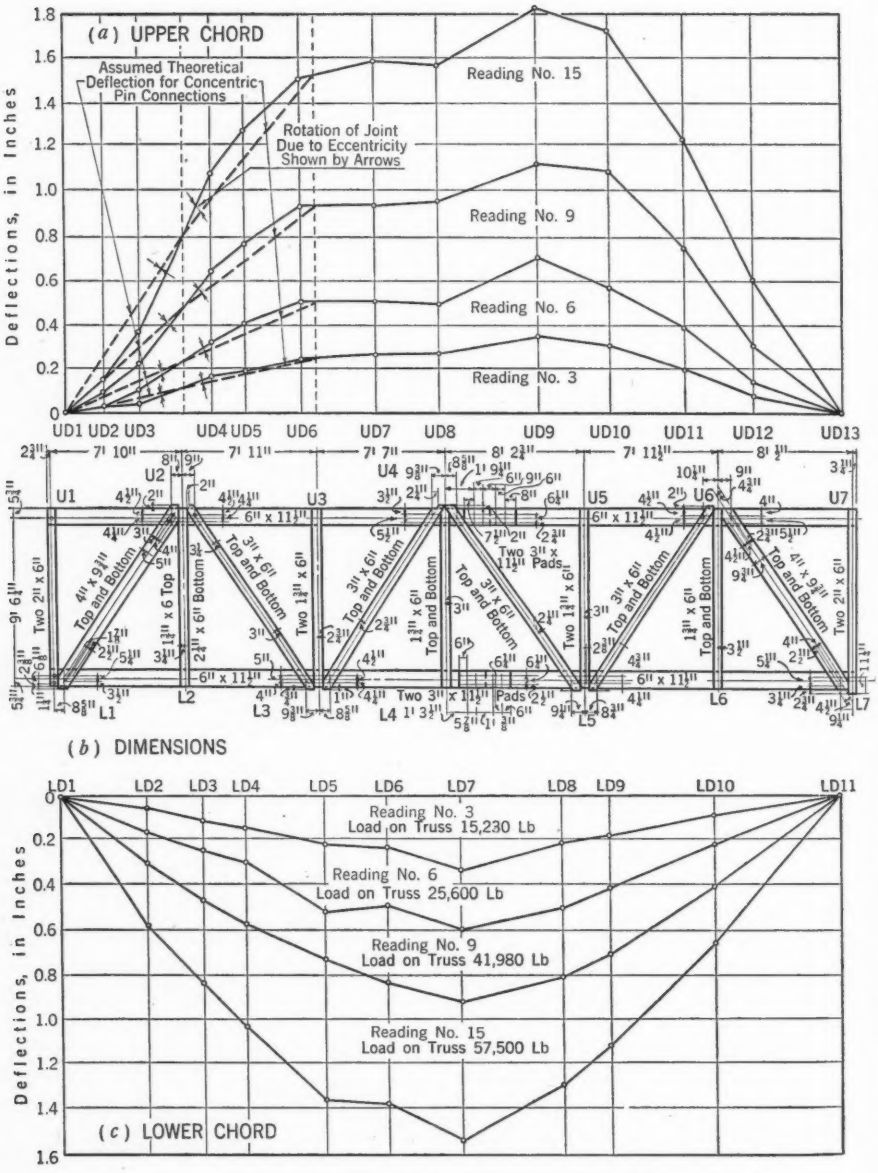

(a) UPPER CHORD
Assumed Theoretical Deflection for Concentric Pin Connections
Rotation of Joint Due to Eccentricity Shown by Arrows
Reading No. 15
Reading No. 9
Reading No. 6
Reading No. 3
Deflections, in Inches
UD1 UD2 UD3 UD4 UD5 UD6 UD7 UD8 UD9 UD10 UD11 UD12 UD13
U1 U2 U3 U4 U5 U6 U7
L1 L2 L3 L4 L5 L6 L7
(b) DIMENSIONS
LD1 LD2 LD3 LD4 LD5 LD6 LD7 LD8 LD9 LD10 LD11
Reading No. 3 Load on Truss 15,230 Lb
Reading No. 6 Load on Truss 25,600 Lb
Reading No. 9 Load on Truss 41,980 Lb
Reading No. 15 Load on Truss 57,500 Lb
Deflections, in Inches
(c) LOWER CHORD

FIG. 38.—LOAD-DEFLECTION CURVES, TRUSS TNS

Angular changes between members as well as slips of connectors were also measured. Additional sets of gages were placed on three of the web members to measure the bending effects caused by direct loads, since these loads were applied at the faces of the members rather than at their centers of gravity.

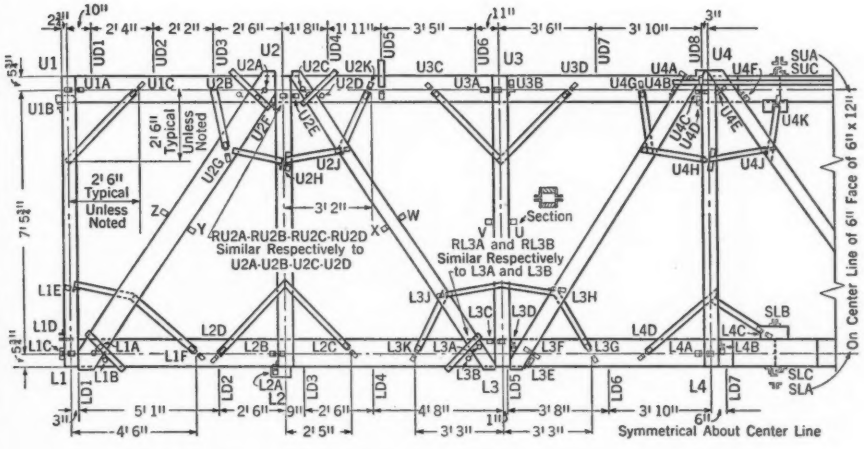


FIG. 39.—HALF-SECTION, TRUSS TNS, SHOWING LOCATION OF GAGE POINTS

A total load on the truss of 42,000 lb was applied gradually over a period of 8 hours. Loads were then released and testing was resumed about 48 hours later. After reaching a load of 62,600 lb a cracking noise was heard and a split opened in the lower chord at joint L3. This split was ½ in. wide, and was increasing in width as the pressure on the loading jacks decreased. Loads were then released and stitch bolts were installed in the chord at joints L3 and L5 as shown in Fig. 37, although the nuts of the bolts were not tightened until the truss refused to take further load.

When the test was resumed, failure developed again at joint L3 at the same load of 62,600 lb. When it was apparent that the truss would not resist greater loads, the stitch bolts were released and the nuts were tightened. Testing was resumed; and, at a load of 65,020 lb on the truss, joint U2 failed, as shown in Fig. 40 (note the uncovered ring groove on the left-hand diagonal). Further testing after reinforcing the joint with stitch bolts brought about deformation in the top chord, as shown in Fig. 41. After further reinforcing joint U2, the next failure occurred at joint U6 in a similar manner at a load of 67,400 lb. It is important to record that, in spite of existing deep checks at joint U6, splitting followed new lines, although at some points they were only ½ in. apart. Fig. 37(*a*) shows the position of splits in the chords which was typical for all three joint failures. The location of the new splits was seemingly independent of existing checks in all cases.

Results of Test Truss TNS.—The results of this test are summarized in Table 6.

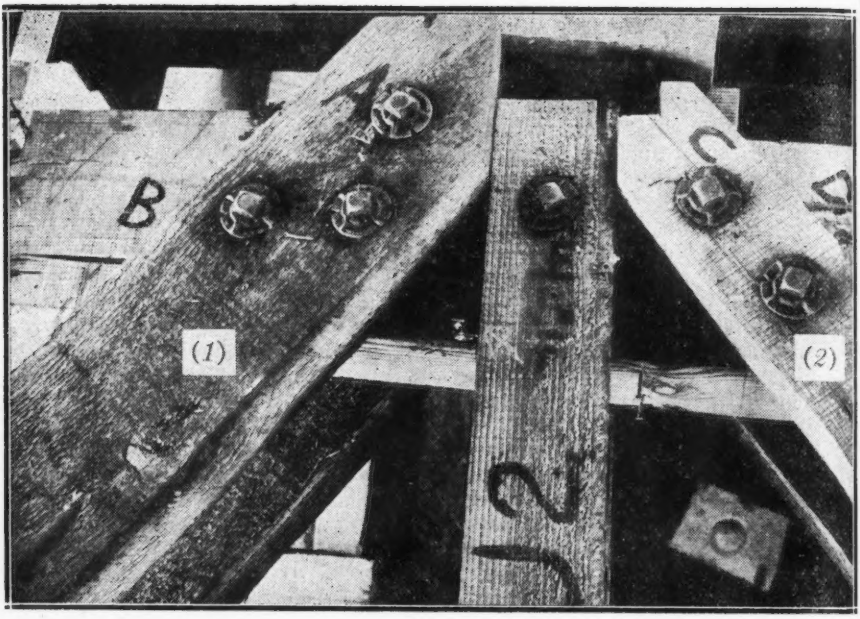


FIG. 40.—FAILURE AT JOINT U2 OF THE NORTH SQUARE COURT TRUSS

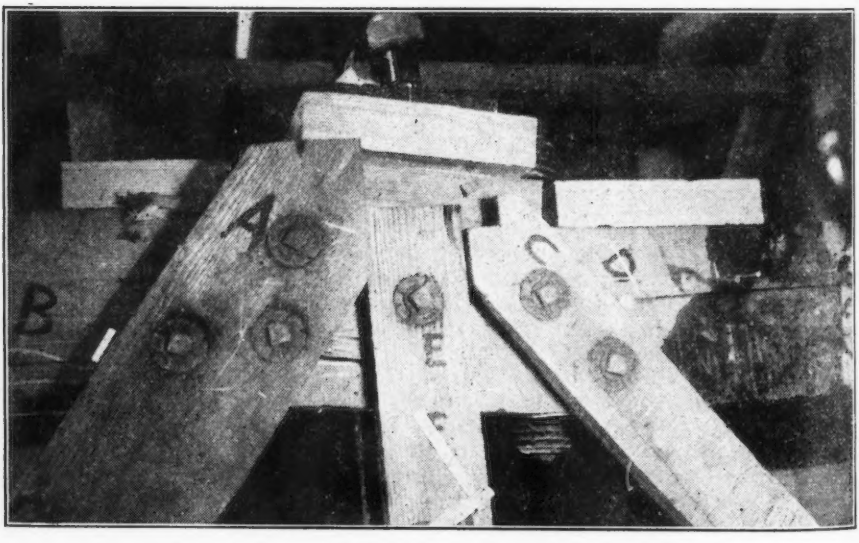


FIG. 41.—FAILURE AT JOINT U2 WITH STITCH BOLTS TIGHTENED

TABLE 6.—STRESSES, ALLOWABLE LOADS, AND RATIOS FOR THREE FAILURES, TRUSS TNS

Description	JOINT L3[a]			JOINT U2[a]		JOINT U6[a]	
	U4–L3	U3–L3	U2–L3	U2–L3	U2–L1	U6–L5	U6–L7
Maximum truss load at failure (kips)		64.6		65.0		67.4	
Stress, F, in the member at failure (kips)	0.9	15.9	24.6	25.5	48.4	22.1	49.6
Allowable load, P (kips)	17.9	7.7	17.9	17.9	27.2	17.9	27.2
Ratio, R_{ca}, at failure	....[b]	2.07	1.37	1.42	1.78	1.23	1.83

[a] After failures shown in table, each individual joint was reinforced to permit further testing and failure at other joints. [b] Not a factor in failure.

The following information concerning deflections was observed:

Item	Description	Quantity
	Deflection at the Allowable Truss Load of 36.5 Kips, Based on an Allowable Stress in Diagonal U2–L3 Corresponding to a Load of 930 Lb per Lin Ft of Truss:	
1	In inches	0.84
2	Ratio of span	1/685
	Maximum Recorded Deflection, at a Truss Load of 57.5 Kips:	
3	In inches	1.56
4	Ratio of span	1/370

Fig. 38 shows the deflections of the bottom and top chords under various loadings. The effect of bending moments in chords, caused by the eccentric connections of the web members, is illustrated by the dotted lines in Fig. 38(*a*). Fig. 42 indicates the center deflections of the bottom chord of the truss at various stages of loading. Fig. 43 gives data on the bowing of the two sticks of diagonal L1–U2, because the loads were applied at their faces instead of at the centers of gravity. This condition occurs quite commonly in timber design, and is generally neglected. As the timbers in such a connection shrink and bolts become loose, the end restraint of the members thus connected becomes less effective with consequent increase in bending stresses. Members composed of two sticks, as were these web members of this truss, would have their load capacity increased at only slight additional expense if their sticks were tied together to form an integral member similar to the "battened" steel column.

Fig. 44 shows the slips of connectors at either end of diagonal member L1–U2. The average slip of a connector (split ring) was about 0.06 in. for a load of 4,000 lb applied at an angle of 50° with the grain of the wood. Fig. 45 shows the deformations at four gage points at joint U2. Using these data, the approximate angular change between the various members may be calculated.

The most important feature of this test was the demonstration that failure in tension perpendicular to the grain in the chord members at the eccentric joints limited ultimate load-carrying capacity. In the history of timber

testing, there seems to be no previously published record of this type of failure. It is known that, when designing this truss, shearing and bending stresses created in the chords by the eccentric connections were carefully investigated; yet, in spite of the unusual care exercised in the design and in spite of unusually

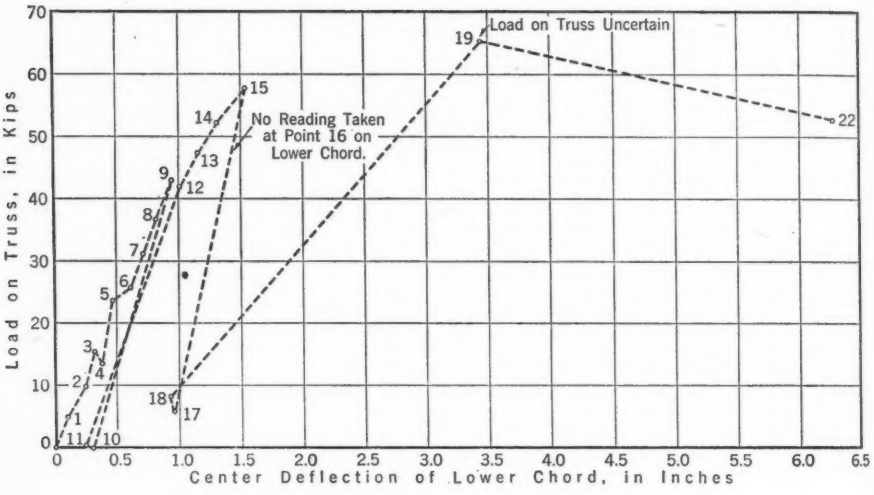

Load on Truss, in Kips
Center Deflection of Lower Chord, in Inches
Load on Truss Uncertain
No Reading Taken at Point 16 on Lower Chord.

FIG. 42.—CENTER DEFLECTION OF LOWER CHORD, TRUSS TNS

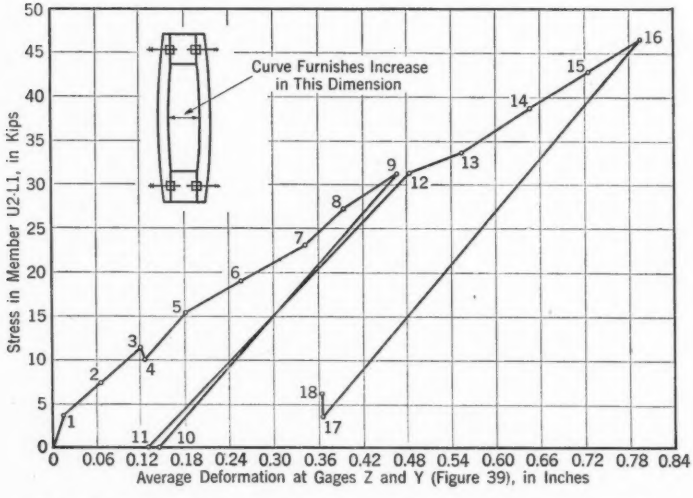

Stress in Member U2-L1, in Kips
Average Deformation at Gages Z and Y (Figure 39), in Inches
Curve Furnishes Increase in This Dimension

FIG. 43.—STRESS-DEFORMATION CURVE AT GAGES Z AND Y, MEMBER U2–L1

good workmanship in construction, the truss failed at a R_{wa}-ratio or a R_{ea}-ratio of 1.78—an unexpectedly low value. This weakness would probably not have been developed in tests on small specimens. Again the vital importance of tests on actual trusses is demonstrated.

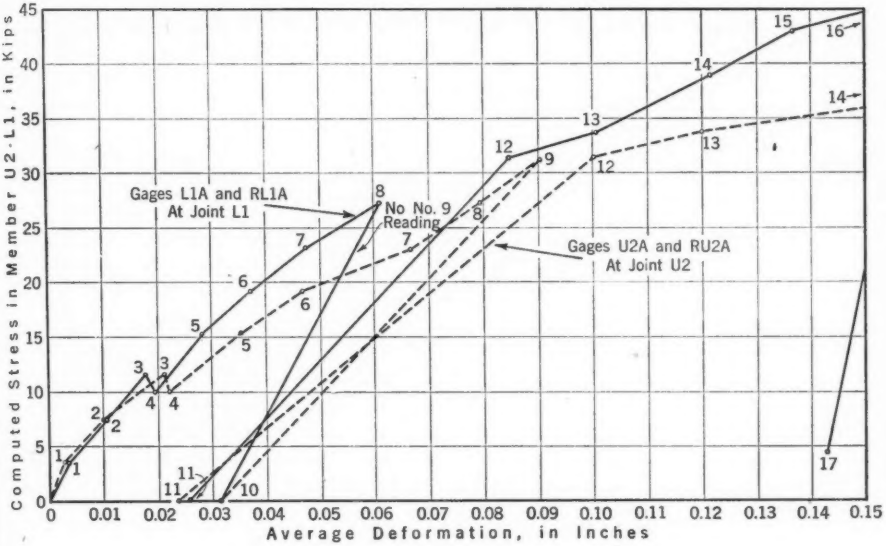

Computed Stress in Member U2-L1, in Kips
Average Deformation, in Inches
Gages L1A and RL1A At Joint L1
No No. 9 Reading
Gages U2A and RU2A At Joint U2

FIG. 44.—SLIP OF THE CONNECTORS ON DIAGONAL L1–U2

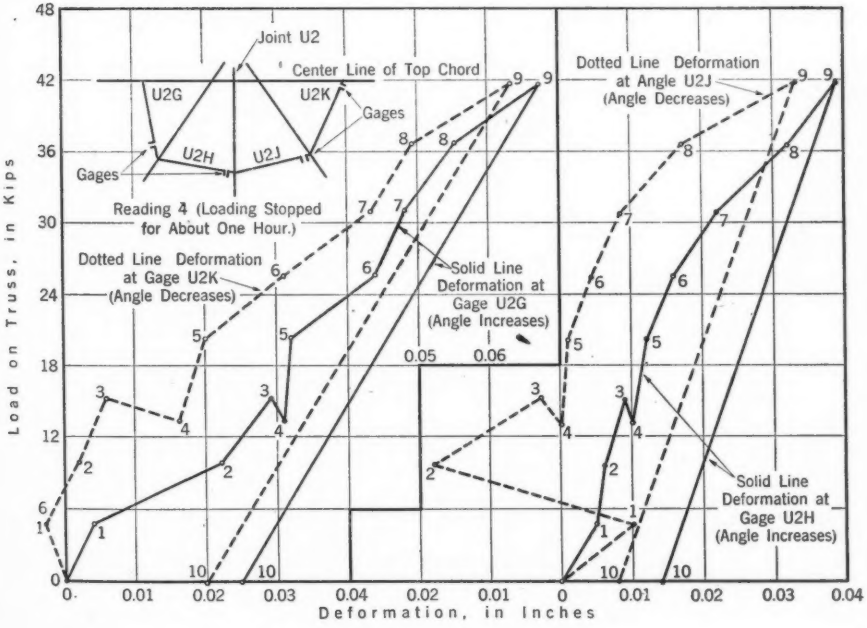

Load on Truss, in Kips
Deformation, in Inches
Joint U2
Center Line of Top Chord
U2G
U2K
U2H
U2J
Gages
Reading 4 (Loading Stopped for About One Hour.)
Dotted Line Deformation at Gage U2K (Angle Decreases)
Solid Line Deformation at Gage U2G (Angle Increases)
Dotted Line Deformation at Angle U2J (Angle Decreases)
Solid Line Deformation at Gage U2H (Angle Increases)

FIG. 45.—DEFORMATIONS AT JOINT U2

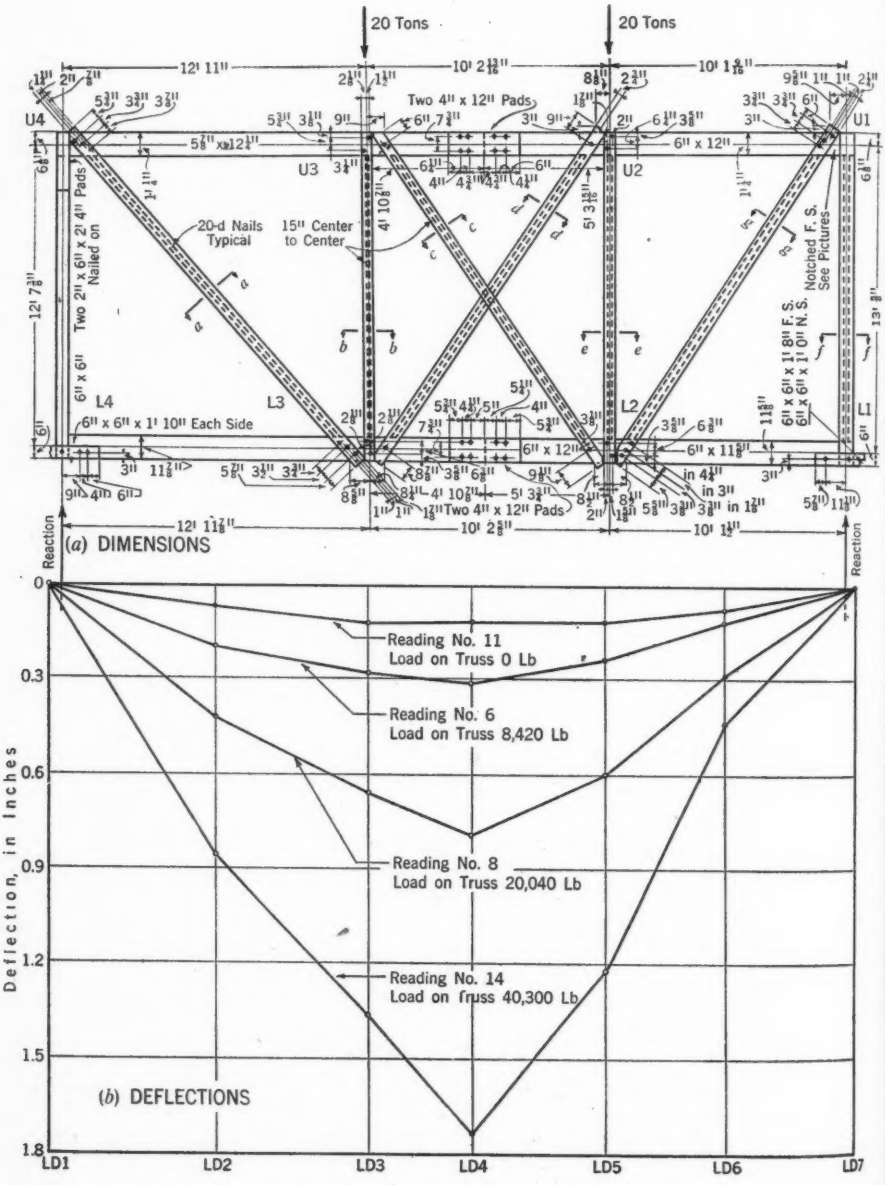


FIG. 46.—DEFLECTION OF LOWER CHORD, TRUSS TSF (FOR SECTION DETAILS, SEE FIG. 47)

San Francisco Building (SF) *Truss.*—This test was of an all-timber truss with bolted connections taken from the San Francisco Building. It had a span of 33 ft 4 in. and a depth of 13 ft 10 in., and its chords were substantially parallel. When tested it had a moisture content of about 14%.

The details of this truss are shown in Figs. 46(*a*) and 47. Web members were I-shaped, made of three 2-in. by 6-in. sticks nailed together. The 2-in. by 6-in. webs were butted against the 6-in. by 12-in. chords of the truss, being angle cut in the base of the diagonals, and the flanges were carried over the sides of the chords and bolted through with $\frac{7}{8}$-in. round bolts with malleable iron washers. Timbers were of select structural grade Douglas fir, and were in good condition except at joint U1, where the end of one flange of the diagonal U1–L2 had been notched to provide clearance for a roof rafter. This flange had a check which had started at the point of the notch. Shrinkage of the wood had caused all bolts to become loose from about $\frac{1}{16}$ in. to $\frac{1}{8}$ in.

Attention is called to the pattern of the bolt arrangement at the end connections of U1–L2 and U4–L3 (see Fig. 46(*a*)). At joint U1, for example, the longitudinal center lines of bolts were staggered by about 1 in. only; and

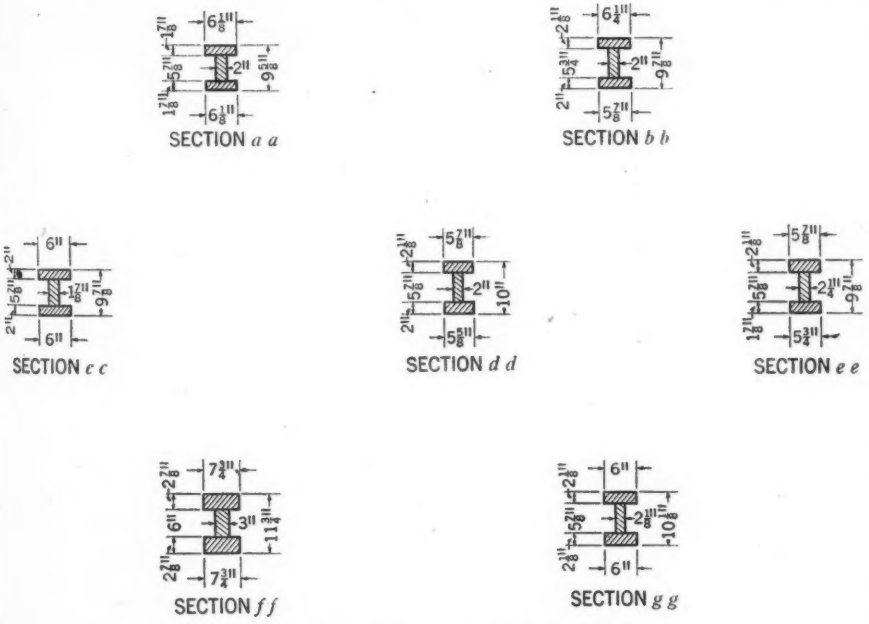


FIG. 47.—SECTION DIMENSIONS (SEE FIG. 46(*a*))

this feature, combined with the notching and checking, proved to be the cause of an early failure of this joint.

Testing.—The truss was installed horizontally in the testing structure, and tested in the usual manner. Testing was started late one afternoon, and loads of 1,840 lb and 1,650 lb were applied to panel points U2 and U3, respectively. Deformations and deflections were measured, loads were released, and

the recovery was measured. The next day, the very slight recovery overnight was measured and loads were again applied. The first failure occurred at joint U1 in the notched end of U1–L2, at a total truss load of 18,700 lb. The damaged 2-in. by 6-in. flange of member U1–L2 was removed (see Fig. 48(*a*)). The block had sheared out by the bolt, and its width is the same as the diameter of bolt. Fig. 48(*a*) reveals no evidence of wedge action.

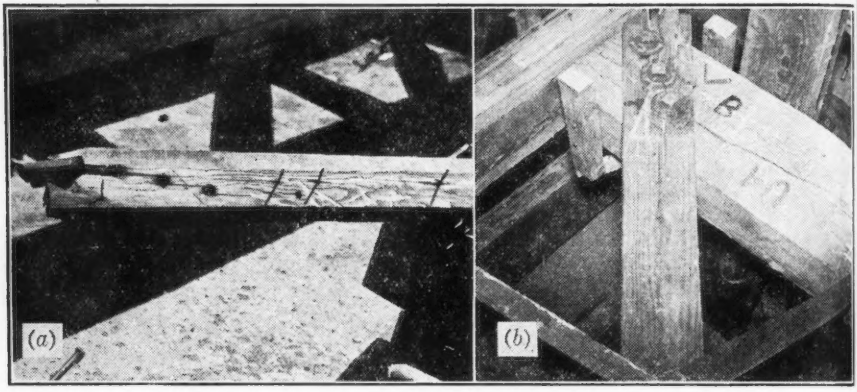


FIG. 48.—FAILURES OF UPPER CHORD JOINTS, TRUSS TSF
(*a*) JOINT U1
(*b*) JOINT U4

After this failure, it was decided to repair the joint and continue testing. This was done on the next (the third) day, and testing continued until failure occurred in one flange of the diagonal L3–U4 at joint U4 at a total load on the

TABLE 7.—SAN FRANCISCO BUILDING TRUSS TSF

Deflection.—At a total allowable load (corresponding to about 375 lb per lin ft of truss, or a total load of 8.2 kips), the deflection at the center of the bottom chord was 0.25 in.—that is, 1/1,600 of the span length. The maximum recorded deflection (with a load of 40 kips on the truss) was 1.70 in.—that is, 1/235 of the span length.

Joint	Stress in End Diagonals					Ratio R_{bs}		Residual stress (lb)
	At Failure[a]			Allowable (Lb)[b]				
	(Kips)	(Lb per lin ft)	Total load (kips)	End bearings	Cross bearing	Col. 2 / Col. 6	Col. 2 / Col. 5	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Joint U1	18.7	1,100	24.0	9,468	6,390	2.94	1.98	15,511
Joint U4								
Part failure[c]	31.9	1,750	38.4	9,000	6,840[d]	4.65	3.54	34,800[e]
Total failure[e]	34.8	1,900	41.6	9,000	6,840[d]	5.09	3.87	19,800

[a] Stress F in the member (Col. 2) corresponding to applied loads of w (Col. 3) or W (Col. 4). [b] Based on the bolts of the given joints being subjected to end bearing (Col. 5, at 0° to the grain) or cross bearing (Col. 6) for stress on the end diagonal. In Joint U1 (6-in. by 12-in. chord) the cross grain was at about 57° and in Joint U4 at about 45°. [c] Only the top member of the I-section failed at 31.9 kips. The loading was increased until at 34.8 kips the entire connection failed—top and bottom. [d] Not critical. [e] The load was increased to a maximum of 34,800 lb.

truss of 31,900 lb. Without repairing the joint, the load was increased until, at 34,800 lb, the other flange of the diagonal at joint U4 also failed. Fig. 48(*b*) illustrates these last failures.

Results of Tests, Truss TSF.—All failures of the truss occurred in the wood at the bolted connections. Fig. 46 shows the deflection curves of the lower chord. Fig. 49 shows the center deflection of the lower chord plotted against

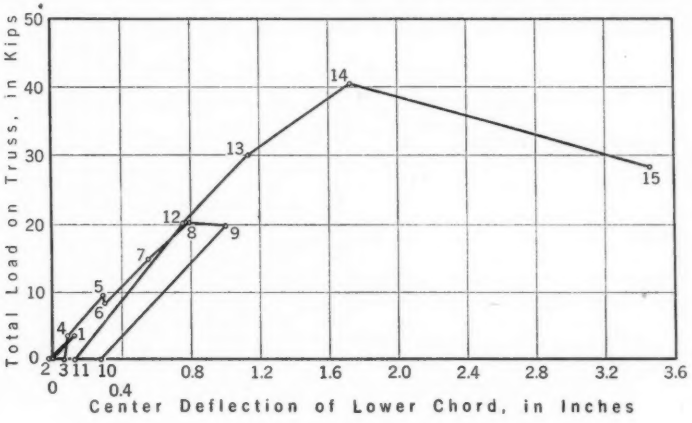


FIG. 49.—CENTER DEFLECTION, LOWER CHORD, TRUSS TSF

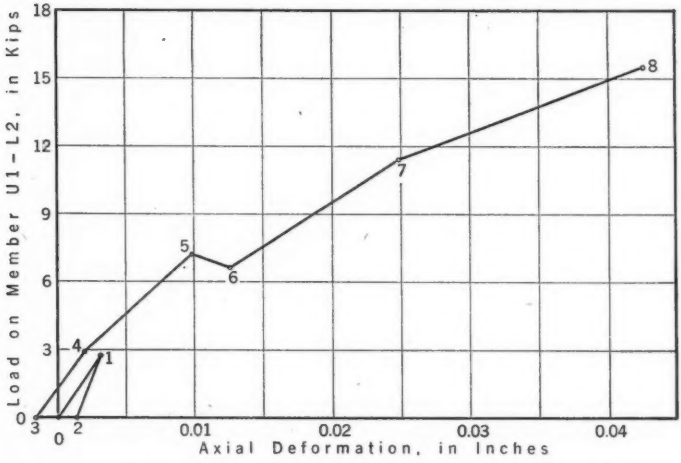


FIG. 50.—DEFORMATION OF U1-L2 WITH RESPECT TO UPPER CHORD TRUSS TSF (CONNECTION CONSISTS OF THREE ¾-IN.-ROUND BOLTS)

the total load on the truss, and Fig. 50 shows the axial deformation of the diagonal member U1–L2 with respect to the 6-in. by 12-in. chord at joint U1.

The allowable load on this truss is that determined by the safe capacity of the joint detail (U1) that failed—in this case determined by the allowable load on the bolts, P_b. The computed load (8,200 lb) corresponds to about 375 lb per ft of truss. If the failure of the defective notched joint U1 is ex-

cluded as subnormal, the truss may be stated to have failed at a load of 31,900 lb, corresponding to about 1,750 lb per lin ft of truss.

It is important to note that the joints did not fail in the manner indicated by calculations of their strength. To illustrate, Table 7 indicates that, in joint U1, the allowable load for the diagonal U1–L2 is the allowable load for the bolts in the end of the diagonal based on bearing parallel to the grain of the two 2-in. by 6-in. diagonals, or P_b would equal 9,648 lb. The critical allowable load of the joint, however, would be based on the bolts bearing at 57° to the grain in the 6-in. by 12-in. chord, or P_b would equal 6,390 lb. Joint U1, however, failed in the diagonals by shearing out the ends of the 2-in. by 6-in. sticks. Basing the ratio R on the action that caused failure, R-values of 1.98 at joint U1 and 3.54 at joint U4 are determined. The greatly superior strength of joint U4 may be ascribed to two factors—first, the bolts at joint U4 were staggered slightly farther apart than those at joint U1; and, second, at joint U1, checks had started at the base of the reentrant angle cut, weakening the shearing resistance of the wood to the bolts.

The arrangement of the bolts in joint U1 was poor, undoubtedly lowering the joint strength. A smaller number of bolts with proper spacing and end distances (in accordance with the recommended practice of *Technical Bulletin No. 332*[4] of the Forest Products Laboratory) probably would have given equal strength, whereas only one pair of 1-in. bolts, with 1½-in. edge distances and 8-in. end distances would have yielded a higher strength than the three bolts with the arrangement and spacings used.

The importance of proper field supervision is emphasized by the unfortunate notching of an important structural member for a very minor purpose—the location of a roof joist. In this case the notch unquestionably reduced the strength of the truss.

Before it was repaired, the truss yielded a $R_{bα}$-value of 2.94, in bearing at an angle to grain, failure parallel to the grain being caused apparently by design and construction defects at joint U1 (see footnote[b], Table 7). After repairs in which these deficiencies were corrected, the carrying capacity of the truss was increased nearly 75%, although the detail of bolting for joint U4 could have been much improved.

An inherent property of timber structures is illustrated in this test: Even after the truss failed, it successfully held a load corresponding to two or three times the allowable load.

Comments on Tests of Trusses.—The following comments, representing conclusions drawn from the results of the tests of the sixteen trusses (twenty tests), seem justified:

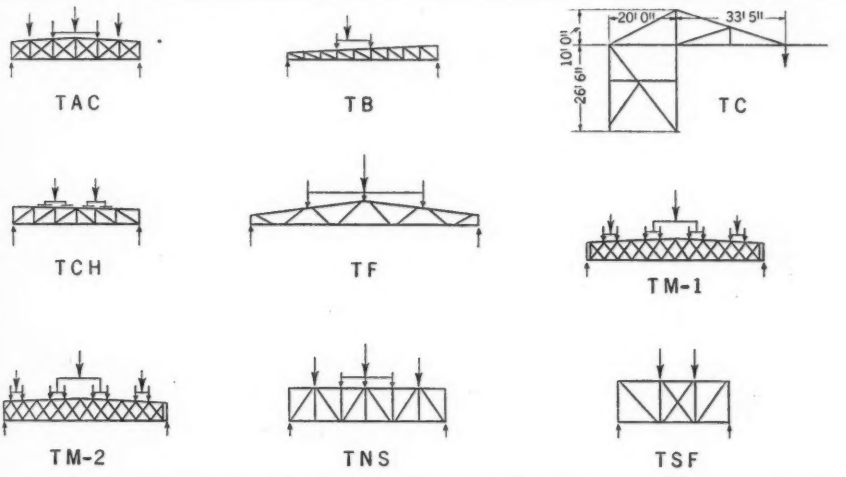
1. Of the twenty tests made, on sixteen trusses, the results show that—five failed because the wood members ruptured at joint details; six failed at bolted joints; and nine failed at connector joints, due to types or combination of stresses not anticipated.

2. The truss with the highest R-value of 7.3 was truss TCh–5 (see Table 8) which had its compression diagonals framed on one side of the chords and its

[4] "The Bearing Strength of Wood Under Bolts," by George W. Trayer, *Technical Bulletin No. 332*, Forest Products Laboratory, Forest Service, U.S.D.A., Madison, Wis., October, 1932.

tension diagonals on the other—a type discussed in conclusion 6. Except for the end diagonals, connections of web members to chords were made with single bolts.

TABLE 8.—SUMMARY OF TESTS, DOUGLAS FIR TIMBER TRUSSES



Test number	R	Deflection: Design load	Deflection: Ultimate load	Type and location of failure
(a) ALAMEDA-CONTRA COSTA BUILDING TRUSSES, TAC; SPAN, 39 FT 4½ IN.; HEIGHT 6 FT 0½ IN.; SELECT GRADE OF TIMBER; ROD-AND-BLOCK WOOD DIAGONALS, AND STEEL ROD VERTICALS				
TAC-1..	3.15	1/790	1/255	Failed by shear in the butt blocks. Low shear values were the result of the shape of butt blocks and the combined stresses.[a]
TAC-2..	2.95	1/665	1/257	
TAC-3..	2.55	1/775	1/378	Failed by shear in the butt blocks. The surfaces of the butt blocks were paraffined thus reducing friction and yielding lower test results.
TAC-4..	2.0	1/775	1/468	
(b) BRAZIL BUILDING TRUSSES TB; SPAN 44 FT, 6½ IN.; HEIGHT 2 FT, 10 IN.; SELECT GRADE OF TIMBER; SPLIT RINGS ON THREE STICK CHORDS AND TWO STICK WEBS				
TB-1....	2.95[b]	1/233	1/107	The chords failed in tension across the grain, TB-2 failing at the center chord splice.
TB-2....	2.48[b]	1/214	1/191	
(c) CAVALCADE CANTILEVER-FRAME, TC; SPAN 33 FT, 5 IN.; HEIGHT 10 FT; NO. 1 TIMBER, WITH SHEAR PLATES AND SPLIT RINGS				
TC.....	....	....	....	The crown joint failed under preliminary loading; careless workmanship.
(d) CHINESE BUILDING TRUSSES, TCH; SPAN 38 FT, 8¾ IN.; HEIGHT 4 FT 4½ IN.; NO. 1 DIMENSION TIMBER; ALL SINGLE STICK MEMBERS WITH BOLTED AND NAILED JOINTS				
TCh-1...	5.3	1/690	1/323	Failure was by tension in the lower chord splice. There was considerable distortion under load because of unsymmetrical framing.
TCh-2...	6.4	1/870	1/141	
TCh-3...	5.7	1/685	1/119	Failure was by bending and direct stress in the end diagonal.[c]
TCh-4...	5.9	1/783	1/169	Bolts sheared out of end diagonal.[c]
TCh-5...	7.3	1/885	1/149	

TABLE 8.—(*Continued*)

Test number	R	Deflection		Type and location of failure
		Design load	Ultimate load	
(*e*) French Building Trusses, TF; Span 35 Ft, 7½ In.; Height 3 Ft, 6 11/16 In.; Select Grade of Timber; Split Rings, with One Stick Chords and Two Stick Webs				
TF.....	1.5[d]	1/720	1/270	Cross-grain failure by tension in the chord at joint L2.[e]
(*f*) Merry-Go-Round Building Trusses, TM; Height[f] 6 Ft; Select Grade of Timber; Split Rings in Two Stick Chords (One System); Diagonals Are One Stick and Two Stick Members				
TM–1...	3.3[g]	1/567	1/173	Failed by chords splitting at several points.[h]
TM–2...	3.60[g]	1/630	1/187	Bottom chord split at joint L2.[h]
(*g*) North Square Court Building Trusses, TNS; Span 47 Ft, 6½ In.; Height 10 Ft, 5¾ In.; Structural Grade of Timber, with Split Rings in the One Stick Chords and the Two Stick Web				
TNS....	1.78	1/685	1/370	Cross-grain failure by tension[i] in the chord at joint L3.
(*h*) San Francisco Building Trusses, TSF; Span 33 Ft, 4 In.; Height 12 Ft, 10 In.; Select Grade of Timber with Bolted Joints and Nailed Members				
TSF.....	2.0[j]	1/1,600	1/235	Bolts sheared at joint U2, emphasizing importance of proper bolt spacing and arrangement.

[a] The results of these tests show a friction value of 0.315 in the butt blocks. [b] In this shallow truss the chord bending effect was ignored in computing the R-values. [c] The bottom chord splice was reinforced. There was considerable distortion under load. [d] R varied between 1.5 and 3.6. [e] Stitch bolts held excellently after first failure, adequately reinforcing joint. [f] Span lengths: TM–1, 54 ft, 1 in.; and TM–2, 51 ft, 4½ in. [g] These values apply after the chord splices had been reinforced. [h] Initial failure at the bottom chord splice which, apparently, had been injured in dismantling. [i] Eccentric joints require check for cross-grain tension. [j] R varied between 2.0 and 5.1.

3. The deflection of trusses was apparently independent of type, the all-timber truss with modern connectors and the truss with butt-block connections for compression members and tension web members of steel having about equal stiffness.

4. The average deflection for parallel-chord trusses with depth-to-span ratios of 1/5 to 1/8 was, for allowable loads, about 1/700 of the span (varying from 1/567 to 1/885).

5. A number of trusses failed at unexpected loads, such failures emphasizing the lack of information on, or knowledge of, the local stresses to be expected in addition to the direct stresses. Even had check computations been made on the design, using conventional methods, some of these weaknesses probably would not have been disclosed, because in many cases customary methods of truss analysis would not have taken into account the stresses that actually caused failure.

6. The necessity of considering tension perpendicular to the grain for correct design of timber joints was one of the outstanding results of these tests. Examples of this need were disclosed in the cases of eccentric connections of truss members, as (*a*) where chord and web members did not intersect at common points, and (*b*) where two web members connecting at a joint or

panel point were framed on the opposite sides of the chords, causing rotation of the chords, bowing of the webs, or both, with resultant splitting of the chord members.

7. The importance of adequate end distances for bolts, proper spacing of bolts, and especially proper arrangement or pattern of the bolts in bolt clusters,[5] was signally illustrated by some of the truss failures.

8. The possibility and effect of combining important stresses should be investigated in some designs. Suitable reductions of allowable stresses should be made if necessary. For example, where shear and flexural stresses occur simultaneously in a timber detail, as in the end butt block of the Alameda-Contra Costa Counties Building trusses (TAC), an assumed working stress in shear of 150 lb per sq in. resulted in a low R_{wa}-value although this unit shear, and even a value of 180 lb per sq in. is allowed in a joint by authoritative recommendations without qualifying limitations for type of detail or presence of other stresses.

9. The high values of R_{ba} were noticeable for bolted connections where the size, spacing, and arrangement of the bolts were carefully selected, although it must be remembered that the design loads recommended for bolts by the Forest Products Laboratory are based on proportional limit loads rather than on ultimate loads.

10. The indentation of the end diagonals of the Alameda-Contra Costa Counties Building trusses (TAC) into the beveled surfaces of their butt blocks indicates the wisdom, in this type of truss, of paying particular attention to the compression areas afforded the ends of diagonal web members, if a balanced design of truss is considered a desideratum.

II. Joint Tests

Joints with Steel Gusset Plates and Shear Plates.—This part of the test program consisted of tests on seventeen joints in which from two to four (but generally three) members were connected together with steel gusset plates and flush-type shear plates. Fourteen of the joints were taken from the stage framing of the Cavalcade, two from the cantilever root trusses of the same structure, and one from the Triumphal Arch. All the joints except one (joint J1), were concentrically connected, or substantially so—that is, the center lines of the members met at a common point.

All joints had been carefully handled during removal and in transportation, so that their strength factors would not be altered by such operations. The wood was in good condition except in a few instances recorded in the summarized results of Table 9 and noted in the following descriptions of the individual tests:

Joint J2.—Although there was some slight checking, the pretest condition of the members of this joint was excellent. Failure was characterized by splitting and shearing.

Joints J3–J7.—The pretest condition of these joints was excellent. Failure was characterized by splitting and shearing.

[5] "Wood Handbook," Forest Products Laboratory, Forest Service, U.S.D.A., Madison, Wis., 1940, p. 136.

TABLE 9.—SUMMARY OF TESTS, JOINTS WITH STEEL GUSSET PLATES

Joint	Dimensions (In.)		End distance, connectors[c] (in.)	P_s (kips)	Maximum Loads (Kips)			Ratios R		Deformation in failed member (in.)	Stress, F, in member after failure	Testing time (min)
	Size of timber[a] (in.)	No. of bolts[b]			One shear plate	Failed member	P_l	R_{cs}	R_{cl}			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
J2....	6×14	3	7	25.1	19.1	114.4	60.0±	4.56	2.38	0.062	88.4	137[h]
J3....	6×10	2	4½	15.1	13.3	53.24	31.0±	3.53	2.05	0.033	10.7	31
J4....	6×10	2	5½	16.4	14.3	57.1	40.9±	3.48	2.49	0.085	6.2	44
J5....	6×6	2	8½	18.1	21.9	87.7	46.0±	4.84	2.54	0.0735	0	25
J6....	6×6	2	8½	18.1	21.8	87.0	59.5±	4.81	3.28	0.030	0	20
J7....	6×6	2	7¼	18.1	22.8	91.2	45.6±	5.04	2.53	0.06	17.2	30
J8....	6×14	3	6	18.1	....	86.83	32.9±	4.77	1.82	0.11	11.5	39
J9....	6×6	2	8½	18.1	21.7	86.9	44.4±	4.79	2.45	0.036	6.1	25
J10...	6×10	2	5	17.1	....	77.7[d]	52.5±	4.54	3.06	0.085	50.6	61
J11...	8×10	2	6	15.5	12.1	48.4	38.1±	3.12	2.46	0.094	5.9	23
J12...	8×10	2	6	15.5	15.8	63.1	39.1±	4.08	2.52	0.094	18.0[f]	30
J13...	6×14	3	7½	25.1	....	126.4[d]	50.0±	5.06	1.98	0.22	....[g]	57[i]
J14...	8×12	2	8¾	18.1	11.3	45.2	0[e]	2.5	....	0.201	10.9	34
J15...	3×10[a]	4	9¾	18.1	13.1	52.5	0[e]	2.91	....	0.056	10.2	54
J16...	3×10[a]	4	10	18.4	13.4	53.8	<37.5[e]	2.93	....	0.095	7.1	37
J17...	2½×5½[a]	4	10½	18.1	....	33.1	0[e]	1.83	....	....	....	8

[a] Single stick members at each joint except J15, J16, and J17, which were double sticks. [b] Bolts were all ¾ in. in diameter except J11 and J12, which were ⅞ in. Except joints J15, J16, and J17 each bolt included two 4-in. shear plates. The three joints mentioned were designed with one 4-in. shear plate on each bolt. After disassembling of joint J8 it was discovered that three shear plates had been omitted from one side. [c] Connectors were spaced uniformly 6 in., center to center, except joint J16 which was 6½ in. [d] In joints J10 and J13, the compression member failed. In the other joints failure occurred in a tension member. [e] No "proportional limit load" evident. [f] Held by the side; not sheared. [g] Did not fail. The joint was holding but deforming. [h] 1 hr 9 min on one day and 1 hr 8 min the succeeding day. [i] Tested for 24 min; then permitted to stand at the design load over one week end and tested for 33 min on Monday.

Joint J8.—The members of this joint were grooved for six shear plates. After disassembly it was discovered that the three plates on one side had been omitted. Failure was by tension, characterized by splitting and shearing on one side.

Joint J9.—The pretest condition of this joint is reported as "good."

Joint J10.—The pretest condition was excellent except for a large natural check in the compression member that failed. This member was a boxed-heart stick, and failure was characterized by shearing, splitting, and bearing. The shear plate slid into the large check.

Joint J11.—This joint came from a cantilever which failed at a relatively low load because of careless workmanship. The pretest condition of the member that failed was excellent except that one shear plate had a hole $\frac{15}{16}$ in. in diameter, whereas the remaining shear plates had $\frac{13}{16}$-in. holes. The threads of the bolts were in bearing on the steel gusset plates and the shear plates. Failure was characterized by shearing and splitting, the bolts being almost sheared through and badly bent.

Joint J12.—The pretest condition of the tension member that failed was excellent. As in joint J11, the threads of the bolts were in bearing on the steel gusset plates and shear plates, and the bolts sheared through at the threaded sections.

Joint J13.—Despite some checking, the pretest condition of the compression member that failed was good. Members 1 and 3 of this joint did not fail. The joint came together and the members butted. The wood adjacent to the shear plates had failed and the bolts were badly bent.

Joint J14.—The tension member that failed in this joint had appeared in excellent condition until it was disassembled. Evidently the workmen had grooved the holes and knocked away the wood between the shear plates before assembling the joint. Failure was characterized by splitting and shearing. Bolts were badly bent; and failure occurred by shearing along old lines where split rings had been installed badly and where chips of wood had been removed.

Joints J15 and J16.—The members of this joint were in good condition, with some checking at the ends of tension members.

Joint J17.—The members of this joint were in excellent condition before testing. There were no checks in the tension members, but there were knots and a slope of 1/12 in the grain that caused bending failure in the wood. The tension member failed by tension and bending of wood. The shear plate did not fail.

Except for $\frac{3}{8}$-in. gusset plates of joints J11 and J12, all gusset plates were $\frac{5}{16}$ in. thick; $\frac{3}{4}$-in. bolts were used in joints J11 and J12, otherwise all bolts were $\frac{7}{8}$ in. in diameter. The bolt holes, in both steel plates and wood members, in accordance with recommendation of the Forest Products Laboratory, were $\frac{1}{32}$ in. oversize.

The members that failed in joints J10 and J17 were structurally defective; but, aside from these joints, the timber was select structural grade of Douglas fir. At the time of testing all members had an average moisture content of from 12% to 15%. Shrinkage had caused the wood members to lose their once snug fit with the steel gusset plates, but no remedial measures, such as tightening the nuts of bolts passing through the connectors, were attempted.

In all joints the holes in the wood members and in the shear plates matched those of the steel gusset plates very well. In all the joints removed from the Cavalcade stage framing (joints J1 to J10, inclusive, and J13 and J15 to J17, inclusive), at least one of each pair of gusset plates had been drilled in the field. In joints J11, J12, and J14, the members had been bored from both sides, but good fits had been obtained.

Testing Procedure.—The manner in which the joints were installed in the testing machine is shown in Fig. 1, and Fig. 51 shows, diagrammatically, the make-up of joints and method of loading. In detail, the load was applied in each test by an hydraulic jack which was placed at the end of one member, usually the chord member and termed member No. 1. A new tension connection was built at one end of one diagonal and the third member was blocked up to resist the induced compression. One exception to this procedure is: In joint J13, all three members were placed in compression; all connectors were 4-in. flush-shear plates; all bolts were $\frac{7}{8}$ in. in diameter except as noted in Fig. 51; and all gusset plates were $\frac{5}{16}$ in. thick except as noted.

Gages were installed to measure deformations of the various members with respect to the steel plates—that is, the slip of gusset and shear plates with respect to the wood members. Other gages were installed to indicate the angular rotation between members. The purpose of these last measurements was to obtain a basis from which the secondary stresses induced in a member when its connection deformed under load might be estimated. None of the

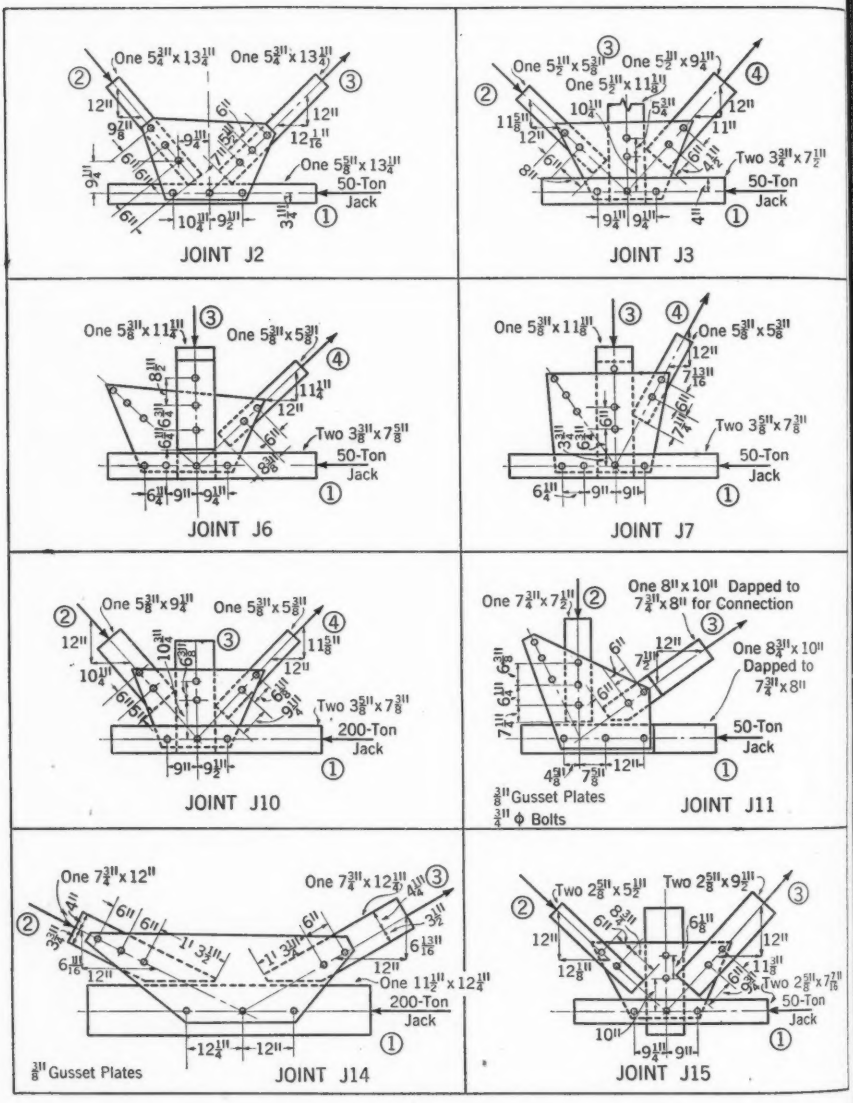
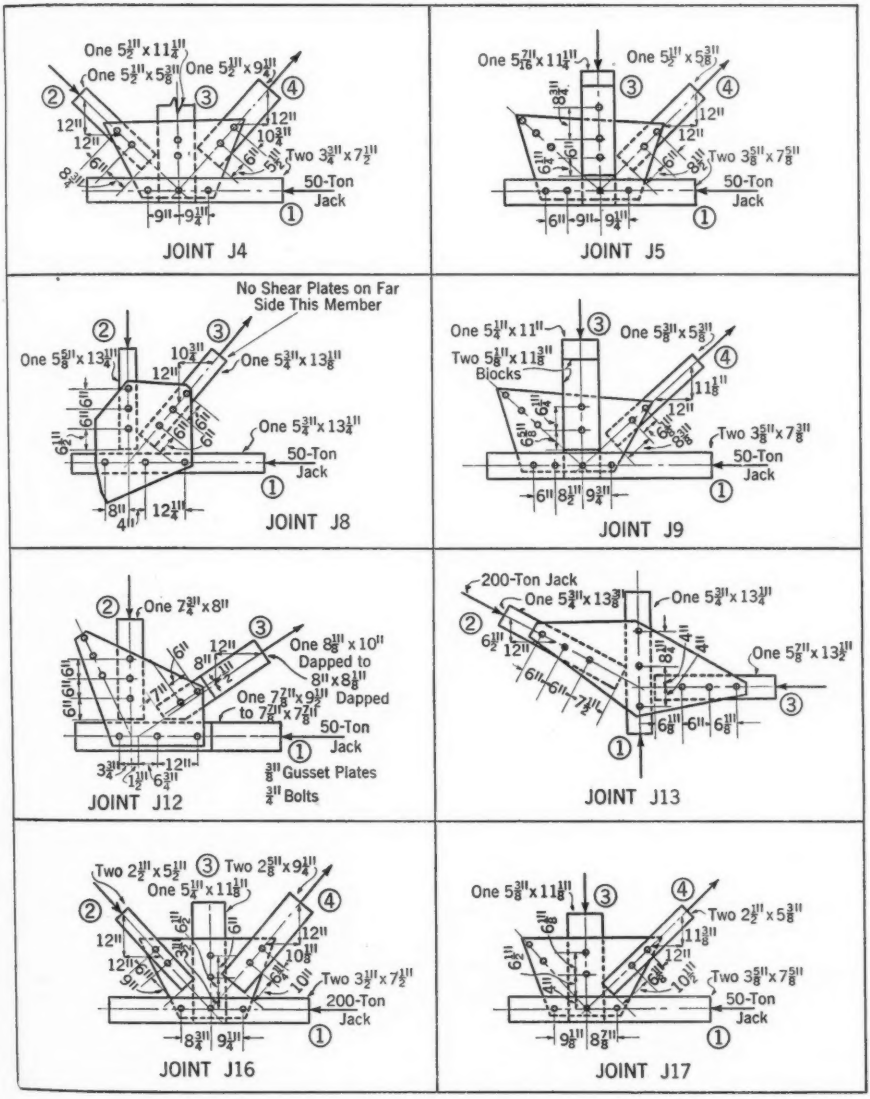


FIG. 51.—MAKE-UP OF JOINTS

bolts in the joints were tightened before testing, and the members of all joints were usually quite loose in their enclosing gusset plates. Because of this condition, all secondary effects and stresses are believed to be negligible and no attempt has been made to evaluate them. Loads were applied in increments of from about 5 kips to 10 kips to approximately the estimated design load; they were then released, the recovery was measured, and the loads were reapplied until failure had occurred in some member or its connection. The



WITH STEEL GUSSET PLATES

gages were generally removed before the ultimate loads were reached. After failure the joints were dismantled, while the results of the test, together with any pertinent observations, were noted and photographs taken. Fig. 1 shows a typical failure in the tension diagonal.

Eccentrically Connected Joint J1.—Fig. 52 shows the detail of this joint. It will be noted that the center line of the diagonal is eccentric by $3\frac{3}{4}$ in. with the centroid of the three pairs of shear plates with its connection to the chord

(member No. 1). In Fig. 52, for convenience of reference, the three pairs of connectors are marked C1, C2, and C3, respectively.

The joint was found to be in excellent condition, with member No. 1, which failed with a moisture content of 13%, a specific gravity of 0.53, twelve rings to the inch, 30% summerwood, and a 1.18 slope of grain. Because of previous experience in testing eccentric joints of trusses in which failure resulted from the stress of tension across the grain, blocks were fastened by

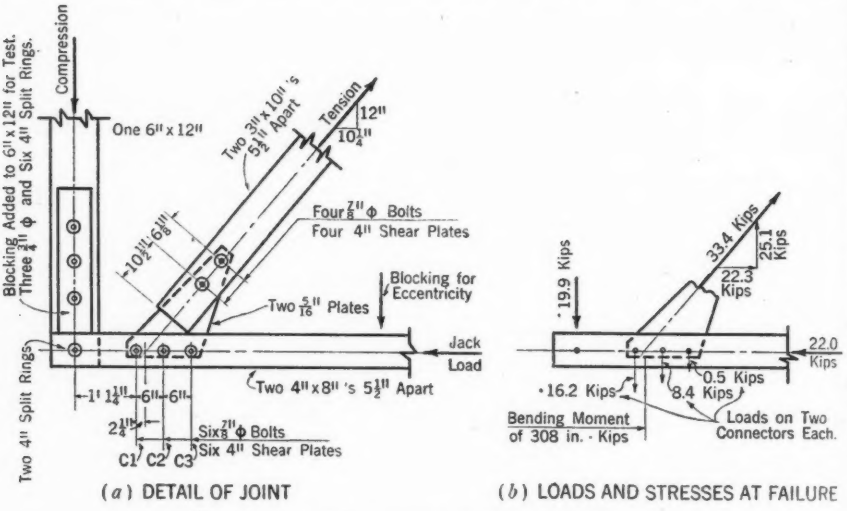


FIG. 52.—ECCENTRICALLY CONNECTED JOINT J1

split rings with bolts to the sides of member No. 2, bearing against the two 4-in. by 8-in. sticks of member No. 1. Also, member No. 1 was blocked laterally to prevent any lateral motion that might have been caused by eccentricity.

As indicated in Fig. 52, a tension connection was provided at the end of diagonal member No. 3 and the joint was installed in the testing truss in the usual manner. Loads were applied through the jack at the end of the two 4-in. by 8-in. sticks of member No. 1 in increments of 5 kips released, and then reapplied until failure occurred. Measurements were taken of the widening of the two sticks of member No. 1 due to tension across the grain. At the left-hand connector of this member, the maximum widening of the member was registered, being 0.01 in., whereas, at the right-hand connector, the widening of the member was only 0.001 in.

Failure occurred through splitting of the two pieces of member No. 1 at a total compression in this member of 22 kips and a simultaneous tension in member No. 3 of 33.4 kips. The following excerpts from the laboratory notes are pertinent:

> "At a load of 16 kips in Member #1 a check in one of the two sticks of this member was opening and a new crack started, while at 20.8 kips the other stick also gave, failing in bending. The first stick went gradually

up to 16 kips and then opened up in tension across the grain following an old check that was about $\frac{1}{2}$" deep. This check kept getting larger until at 20.8 kips the other stick failed in flexure. The first stick failed by bending in the portion that had been split off due to tension across the grain. Both of the sticks also failed by twisting due to the connection of the gusset plates to only one side of each member.

"It was noted that the sticks of Member #3 bent due to the connections of the gusset plates on their sides. This caused the ends of the members to lift off about $\frac{1}{8}$" or more from the gusset plates. Both sticks showed the twisting, or rotational, action of the pull on the inside of members, causing a twist or rotation at failure of an amount sufficient to lift or lower the inside edge of the members about $\frac{5}{8}$" to $\frac{3}{4}$"."

Results of Tests, Joint J1.—The tension in member No. 3 at failure produced an axial stress of 22.3 kips in member No. 1 (due to the eccentricity of the connection), and a transverse shear of 25.1 kips. The axial stress may be assumed to have been resisted equally by the three pairs of shear plates in the member, or by a shear of 7.4 kips per pair of connectors. Because of the eccentricity of the connection (which was really doubly eccentric, since the chord was of two pieces, each connected by one face only), the total transverse shear was not carried equally by the three pairs of connectors. Computations indicate that the left-hand pair of connectors, C1, took a transverse shear of 16.2 kips; and the center pair of connectors, C2, took a transverse shear of 8.4 kips, whereas the transverse shear in the right-hand pair of connectors, C3, was 0.5 kips. Consequently, the maximum resultant shear was found in connectors C1, equal to 17.8 kips, and at a slope of 66° with the grain of the member. The stress P_c on a pair of shear plates, for nondense structural grade of Douglas fir, at an angle of 66° to the grain, is 8,250 lb. Therefore, R_{ca} was $\frac{17.8}{8.25}$, or only 2.16, instead of a value ranging from 3.5 to 5 as was found in the concentric joints of the series. This illustrates the loss of efficiency occasioned by the eccentric connection.

The test emphasizes the fact that the resistance of a joint of this type is lowered by an eccentric connection: First, because of the increased load and increased transverse shear on one or more connectors (in this case 1.55 times the average) resulting from the eccentricity; and, second, because of the increased intensity of stress in tension perpendicular to the grain.

At failure in the two 4-in. by 8-in. pieces of member No. 1, bending stress of 4.90 kips per sq in. occurred. Based on this bending stress, R_{wa} would be 3.04, this value of R having little meaning because failure was not caused by bending. The twisting, or rotation, of the two sticks has been mentioned in the excerpt from the laboratory's notes. This twisting, or rotation, was caused by the steel gusset plates pulling on the sides of the members rather than through their centers of gravity. This phenomenon may be seen in Fig. 53 where the near face of the 4-in. by 8-in. sticks has spread about $\frac{5}{8}$ in. The bending effects caused by plate connections applied on one side of a member is further illustrated in the same photograph where, at the left, the ends of the 3-in. by 10-in. sticks of diagonal member No. 3 are shown separated from the steel gusset plates, which the sticks had caused to bend.

Concentrically Connected Joints.—The details of the concentrically connected joints are illustrated in Fig. 51. Stress ratios, R, are given in Table 9, Cols. 9 and 10.

Joint J2.—Shrinkage of the wood caused the members entering this joint to separate from the gusset plates a distance of $\frac{3}{16}$ in. to $\frac{1}{4}$ in.; and, during

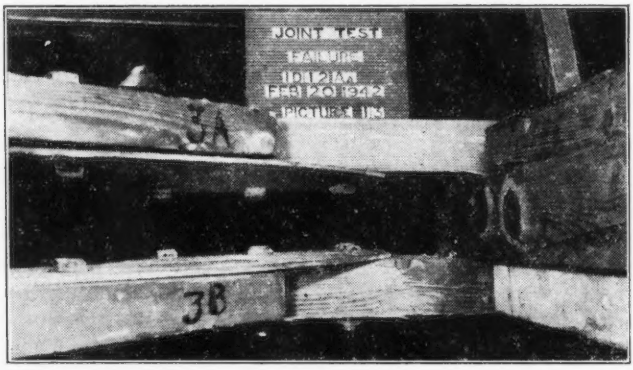


FIG. 53.—EFFECT OF CONNECTING STEEL PLATES ON ONLY ONE SIDE OF EACH MEMBER, JOINT J1

this test, this effort resulted in a buckling of gusset plates. At a load of 42.5 kips in member No. 3, the bolts were bending and the gusset plates at member No. 1 were noticed to be buckling at a load, in the member, of 88.5 kips. Failure occurred in member No. 3 by shearing and splitting of the wood at a total stress of 114.4 kips.

Member No. 1, with six shear plates at 10-in. spacing, resisted without failure or apparent distress 24.6 kips per shear plate, giving a R_{cc}-value of 4.45. This high value of R may be regarded as substantiating the standard spacing of 9 in. recommended for shear plates, although it is to be noted that the gusset plate on one side of member No. 1 had moved laterally 0.11 in. with respect to the member whereas the relative movement on the other side was 0.40 in.—an average of 0.26 in. The large difference in the relative movements of the two faces is difficult to explain, for there was no evident lateral eccentricity of the joint. The following values based on ultimate loads taken by member No. 3 are of interest:

Ultimate load	114.4 kips
Proportional limit load	60.0 kips
Ratio R_{cc}	4.56

Joint J3.—At a total axial stress of 53.1 kips in the tension diagonal member No. 4, this member failed by shearing and splitting, a large split occurring through the bolts and shear plates. Failure went through a deep check. Previous to this failure, and at a stress of 46 kips in this member, a large cracking had been heard and the load had fallen off. The end distance in member No. 4 was only $4\frac{1}{2}$ in.

Joint J4.—Although, in this joint, the tension diagonal, member No. 4, failed at a total axial stress of 57.1 kips, with typical shearing and splitting, a new split, about 3½ in. deep, had occurred in the end of the member at a stress of 24.2 kips. This split was located in the center of the member, parallel to the center line of the shear plate.

Joint J5.—Unlike the most of the other joints this joint failed suddenly. Member No. 4, the tension diagonal, failed by typical shearing and splitting at a total stress of 87.7 kips. On dismantling the joint, it was noted that

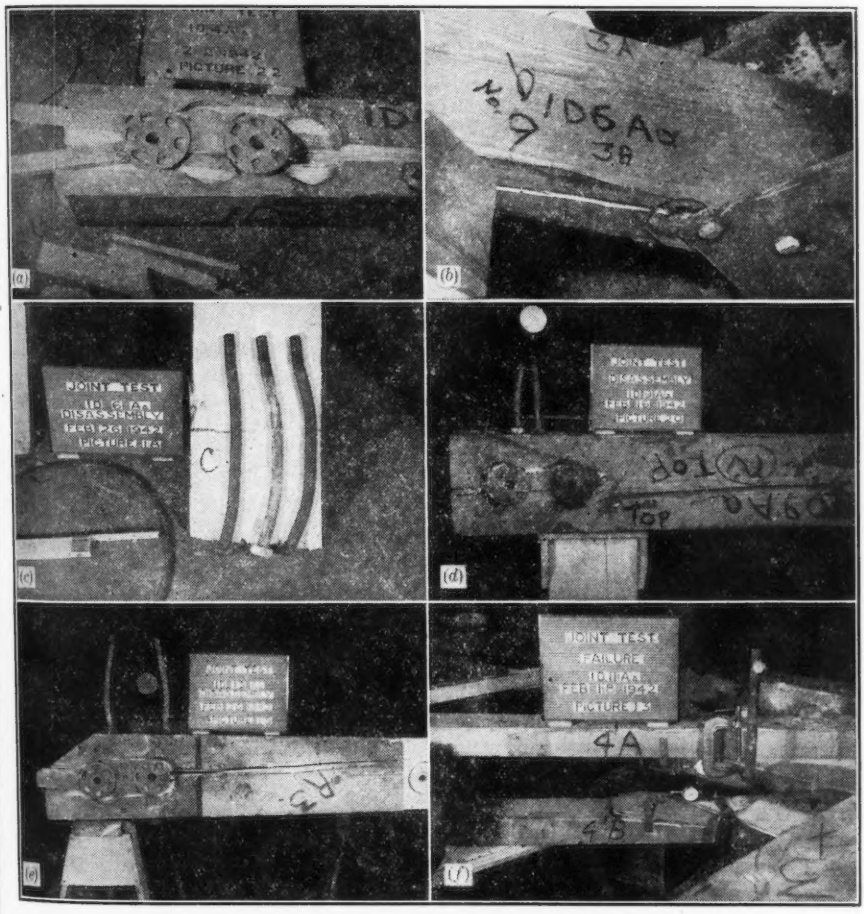


FIG. 54.—THE FAILURE OF CONCENTRICALLY CONNECTED JOINTS

(*a*) JOINT J5
(*b*) JOINT J8
(*c*) JOINT J8
(*d*) JOINT J10
(*e*) JOINT J11
(*f*) JOINT J17

member No. 3, in compression, showed no distress and that it was well fitted. At the time that member No. 4 failed, each shear plate in member No. 3 had resisted 10.2 kips. The end distance was 8½ in. (Figs. 1 and 54(*a*)).

Joint J6.—The test behavior of this joint was similar to that of joint J5. The end distance was $8\frac{1}{2}$ in.

Joint J7.—Member No. 4, the tension diagonal that failed, did not fit tightly between the gusset plates, because of shrinkage which was estimated to be about $\frac{1}{16}$ in. This member failed, with typical shearing and splitting, at 91.2 kips. Its end distance was $7\frac{3}{4}$ in.

Chord member No. 1 consisted of two $3\frac{5}{8}$ in. by $7\frac{3}{8}$ in. sticks, each connected to its gusset plate on one side only, but each stick was fastened with four shear plates. Although its connections were such as to induce rotation, there was no evidence of distress in this member at the maximum load of 44 kips (5.5 kips per shear plate), probably due to the lateral support given by member No. 3, which extended well into member No. 1, between the two sticks.

Joint J8.—This joint behaved peculiarly during its test, giving much evidence of defective construction. After completion of the test, it was taken apart, when it was found that the three shear plates on one side of member No. 3 had been omitted, although their grooves had been cut in the wood. This omission caused the shear plates on the other side of the member to carry almost the entire load. Fig. 54(*b*) shows one of the shear plate grooves without its shear plate, and Fig. 54(*c*) shows the bent bolts with their evidence of excessive bearing from the shear plates. Nevertheless, the actual capacity of the joint, as determined by the test, when compared to the theoretical capacity as designed, assuming the joint to have had all its designed shear plates, would give $R_{ca} = 3.61$, and $R_{cl} = 1.37$.

Joint J9.—This joint gave visual and audible warning of failure at an axial load in member No. 4 of 82.5 kips, the failure of member No. 4 occurring at 87 kips. At this ultimate load, however, there was no visual crack. The laboratory notes record that the failure "came with a bang."

Joint J10.—This joint was defective, in that the compression diagonal, diagonal member No. 2, had a boxed heart, although when examined before testing, the joint was pronounced to be in excellent condition; however, it was observed that member No. 4 had a large check. The laboratory notes record:

> "The check in this member opened about $\frac{1}{2}$" and split all the way through to the other side, where a new split about $\frac{1}{2}$" wide was opened. This tension member, Member #4, did not fail, there being no indication of any timber distress. No cracks on the top, bottom, or end of the member were visible, but the stick had moved about $\frac{5}{8}$" from the steel gusset plates and, after disassembling, it was seen that the bolts were badly bent. Cores and wood between the shear plates had sheared, although not in the end distance area. Member #2 was as expected; the cores and the wood between shear plates had sheared. (This member had withstood 77.7 kips, or 19.4 kips per connector.) Bearing marks of the shear plates were in evidence on all bolts but the shear plates were not damaged (see Fig. 61 [54(*d*)])."

In this joint, the 6-in. by 6-in. member which was in tension, resisted without failure a load of 111.4 kips on two pairs of 4-in. shear plates, spaced at 6 in., with an end distance of 9 in. Thus, each shear plate resisted a load of 28 kips, equal to six and one half times its allowable load.

This member was a single stick, with its connection in double, instead of single, shear; and, although the spacing of connectors was only 6 in., the end distance was 9 in.—the two important factors leading to the high strength. The test of this joint has led to the suggestions, given subsequently under the heading, "Comments on Tests of Joints with Steel Gusset Plates," that: (1) Perhaps the reduced values of allowable connector loads for 6-in. spacings are not justified; and (2) the allowable loads for connectors in single shear should be less than one half of the allowable loads when the connectors are in double shear.

Joint J11.—This joint, taken from the south Cavalcade cantilever truss (which had failed under preliminary test due to poor workmanship) was defective. Upon dismantling it was found that one shear plate had a $\frac{15}{16}$-in. hole for a $\frac{3}{4}$-in. bolt and that both shear and gusset plates bore on the threads of the bolts. Fig. 54(*e*) shows failed member No. 3 after disassembly.

Joint J12.—Fig. 51 illustrates that joint J12 had $\frac{3}{8}$-in. instead of $\frac{5}{16}$-in. gusset plates. The joint was obtained from the north Cavalcade cantilever truss (not the truss that had failed) in which it formed the top detail on the rear end of the anchor arm of the cantilever. To all appearances the joint was in excellent condition, having no important checks or cracks and having undergone but small shrinkage—probably the maximum shrinkage was $\frac{1}{32}$ in.

At a total axial stress of 63.1 kips, member No. 3 failed in its connection. The bolts, which were $\frac{3}{4}$ in. in diameter, sheared off at the nuts, at the root of the thread, and were indented at the ends near the head. The member did not split; neither were there any checks or cracks in the wood of the member. The wood did not shear. After dismantling the joint, it was found that the gusset and shear plates bore on the bolt threads, that a number of shear plates with $\frac{15}{16}$-in. holes had been used with $\frac{3}{4}$-in. bolts, and that these bolts had been bent about $\frac{3}{8}$ in.

Joint J13.—In this joint all members were tested in compression. The joint was in excellent condition with the bolts giving evidence of about $\frac{1}{4}$-in. shrinkage of the wood. The test continued over two days. On the first day, the load on member No. 1 reached a maximum of 15.8 kips. Then, because of mechanical difficulties, the test was stopped. The load automatically dropped to 10.0 kips, and the joint with this load was allowed to stand overnight. On the following day, the load was reapplied, reaching a maximum of 59.2 kips in member No. 1 and 112.1 kips in member No. 3. At a load of 80.4 kips in member No. 2, this member was splitting and (see Fig. 51) the gap between the end of member No. 3 and the side of member No. 1 had closed. With further increase in load, member No. 2 continued to split. The gusset plates were seen to be bowing slightly at the maximum load on the joint (126.4 kips in member No. 2). The shear plates in this member had sheared the wood, and its bolts were badly bent. Because of the bearing of the ends of member No. 2 and member No. 3 against the sides of member No. 1, no direct failure had occurred—that is, the load did not diminish, although it could not be increased (see Table 9).

Joint J14.—As in joint J12, the gusset plates were $\frac{3}{8}$ in. instead of $\frac{5}{16}$ in. The joint was seemingly in excellent condition, the bolts indicating a shrinkage

of the wood from $\frac{1}{8}$ in. to $\frac{1}{4}$ in. Although the structure of the Triumphal Arch from which the joint was obtained had been wrecked by falling, the joint gave no visible evidence of having been damaged.

As the load was applied, deformations of the tension diagonal member No. 2 seemed to be quite large; and, at an axial stress of 29.75 kips in member No. 2, the shear plates in this member appeared to have moved about 0.20 in. Distress of the joint was audible. At a stress of 47.8 kips in member No. 2, the split in the member became wider, but the failure of the joint occurred in member No. 3 at a stress of 45.2 kips, by splitting and shearing of the wood by the shear plates.

On disassembling the joint it was found that the wood cores of the shear plates had been broken and sheared off, stains indicating that this damage had occurred prior to the test. Further evidence of prior damage was the fact that the wood between the shear plates on one side of the connection of member No. 3 and part of the wood between the shear plates was missing. The other part was loose and its edges stained, as if the joint had weathered after the wood had sheared off.

Joint J15.—The joint was in good condition except for a check in one stick of member No. 2 and a small check in one stick of member No. 3. Failure occurred in the tensional diagonal (member No. 3) at an axial stress of 52.5 kips by typical shearing of the wood in the end distance and between the shear plates, combined with the usual splitting. This member was not of solid section, but was composed of two sticks, each connected by one side only; hence, $R_{ca} = 2.91$.

Joint J16.—The joint was in excellent condition except for a check in one of the sticks of tension diagonal member No. 4, the check extending from the last shear plate to the end of the timber. The joint was opened $\frac{1}{8}$ in. before testing. Failure occurred suddenly in member No. 4 by the usual shearing and splitting. As in joint J15, note the lower strength of the two stick member, in that $R_{ca} = 2.93$.

Joint J17.—Before testing, the joint appeared to be in excellent condition, shrinkage of the wood being small. Failure occurred in member No. 4 at a stress of 33.1 kips, one of the two sticks of member No. 4 failing in combined bending and tension (see Fig. 54(*f*)). This member was found to have a 1/12 slope of grain, and a knot at the point where the split at failure intersected at the edge of the stick. The gusset plate bent badly as the member could not take the bending moment induced by its eccentric connection.

Comments on Tests of Joints with Steel Gusset Plates.—The foregoing results lead to ten broad conclusions, as follows:

(1) The necessity of careful and continuous inspection during construction, if material, workmanship, and resultant strength commensurate with the design are to be expected, is particularly illustrated by the tests of these joints. For example, joints J8 and J14, before testing, gave the appearance of being entirely normal. Indeed, an extract from the laboratory notes of the test of joint J8 reads—

"The test of this joint was to have been one of the most important of the Test-Program because of the very excellent condition of the joint and its careful handling prior to the test. However, during the test the behavior of the joint was so peculiar that it was suspected of being deficient internally, and after the test it was taken apart. Then only was it found that the joint had but one-half of its indicated number of shear-plates, those on one side having been completely omitted."

Joint J14 was also found to have been damaged in fabrication; joint J10 had a member with boxed heart with resultant checking; in joints J10, J11, and J12, short bolts had been used, causing gusset and shear plates to bear on the bolt threads; and joint J12 actually failed by shearing of the bolts. Also, in joints J11 and J12 shear plates were used with the wrong size of bolts.

(2) Even with the defects described in item (1), joints J8 and J14 did not fail until they had sustained twice the allowable connector loads P_c for 9-in. spacings and end distances and two and one-half times the allowable loads for 6-in. spacings and end distances.

(3) Effects that can occur due to the presence of boxed heart, knots, or sloping grain are illustrated by joints J10 and J17, in which one or more of these characteristics influenced or contributed to failure.

(4) In a two-piece member each piece is connected on one face only, or with the connectors of each piece in single shear, whereas a one-piece member has connectors in both faces—that is, a double-shear connection. The decreased capacity of the connection of a two-piece member is illustrated by comparison of the average maximum loads at failure of joints J15 and J16 (13.20 kips per shear plate) with the average of joints J2, J5, J6, J7, and J9 (21.6 kips per shear plate). These joints all had end distances well in excess of 6 in. The relative efficiency of the one-piece member with double-shear connection as compared with the two-piece member with single-shear connection raised the question as to whether joints in single shear may properly be considered as having one half of the strength of joints in double shear.

(5) The use of a $\frac{3}{4}$-in. bolt with a 4-in. shear plate, having a $\frac{15}{16}$-in. hole intended for use with a $\frac{7}{8}$-in. bolt, would seem to be a comparatively easy oversight on the part of the carpenter. Some method of avoiding this condition should be developed.

(6) In these tests no $\frac{7}{8}$-in. bolt used with a 4-in. shear plate failed in shear.

(7) The increased capacity of one-piece tension members in double shear, with end distances of 8 in. or 9 in. over one-piece tension members similarly connected but with end distances of only 6 in. or less, was in the ratio of from 1.4 to 1.0—an increase in capacity of 40%.

(8) The slip of shear plates at the allowable or working load was found to be from about 0.05 in. to 0.06 in., confirming published values.

(9) In view of the comment in items (4) and (7), it will be enlightening to group eight of the joints, J3, J4, J5, J6, J7, J9, J15, and J16 (all having 6-in. center-to-center spacings of connectors, but with varying end distances) into three divisions governed by end distance and type of shear and then to note the average ratio value of R_{ca} of each group. Such a grouping is given in Table 10. The joints with connections in single shear, even though they had

ample end distances (group 1), had low values of R. The joints with connections in double shear, even with substandard end distances (group 2), showed increased R-values; and the joints with connections in double shear and with greater than standard end distances (group 3) had the highest values of R. If the nine tests reported in Table 10 are judged sufficient in number to warrant drawing conclusions it would appear that, in this type of joint—

TABLE 10.—EFFECT OF END DISTANCE AND TYPE OF SHEAR

Group	Joints	End distance (in.)[a]	Average R_{ea}
1[b]	J15 and J16	10	2.92
2[c]	J3 and J4	4.5 to 5.5	3.50
3[c]	J5, J6, J7, and J9	7.75 to 9	4.87

[a] Connector spacing 6 in., center to center. [b] Single shear. [c] Double shear.

a. Shear plates at connections in single shear seem to be considerably less effective than such connections in double shear, at least for relatively thin members (3 in.). This condition may be due to stress caused by eccentricity, shrinkage, or other factors. In some cases, the allowable load on a connector in a double-shear connection should be materially greater than that on a connector in a single-shear connection.

b. The reduction in allowable loads for connectors with a spacing of 6 in. center to center, but with a 9-in. end distance, is too severe.

c. The 7-in. end distance for full allowable load for connectors now recommended as standard, as against the former recommended standard of 6 in., is amply justified. Further increase of end-distance requirements might result in even higher allowable loads.

(10) A comparison of the ratios obtained from the joint tests which failed in the shear plates, joints J2 to J9, J12, J15, and J16, is afforded by the data in Table 11(*a*). Joints J15 and J16 were those in single shear which failed at lower ratios than the others. Without these two joints the comparison would be as shown in Table 11(*b*). The joints were fabricated from unseasoned material and tested after seasoning in service. For this condition the recommended working stresses in the TECO Manual of 1939[1] require a reduction in stress of 20%. Calculated on this basis the test results would give comparisons 25% higher than those shown in Table 11.

TABLE 11.—COMPARISON OF EXPECTED AND R-VALUES FROM TESTS

Source	Expected values	(a) JOINTS J2 TO J9, J12, J15, AND J16		(b) JOINTS J2 TO J9, AND J12	
		Average	Range	Average	Range
Manual, TECO[a]	3.5	4.16	2.91 to 5.04	4.43	3.45 to 5.04
Basic values, FPL	4.0	4.78	3.55 to 5.80	5.09	4.00 to 5.80

[a] "Timber Connector Construction," Manual of the Timber Eng. Co., Inc., Washington, D.C., 1939. Forest Products Laboratory, Madison, Wis.

However, it should be noted that in these tests the load was always parallel to grain, under which condition the effect of fabricating with green timber and

testing after seasoning in place is much less severe than when loads are applied at an angle to grain.

Joints with Plywood Gusset Plates, Bolted or Split-Ring Connections.—Joints J18 to J22, inclusive, consisted of one bolted joint connected with a plywood gusset plate and four other joints in which a member was spliced with plywood splice plates fastened with split rings.

The first joint (J18) was removed from the San Francisco Building where it had served as part of the lateral bracing system of the lower chord of the roof trusses. The details of the joint before testing are shown in Fig. 55. The

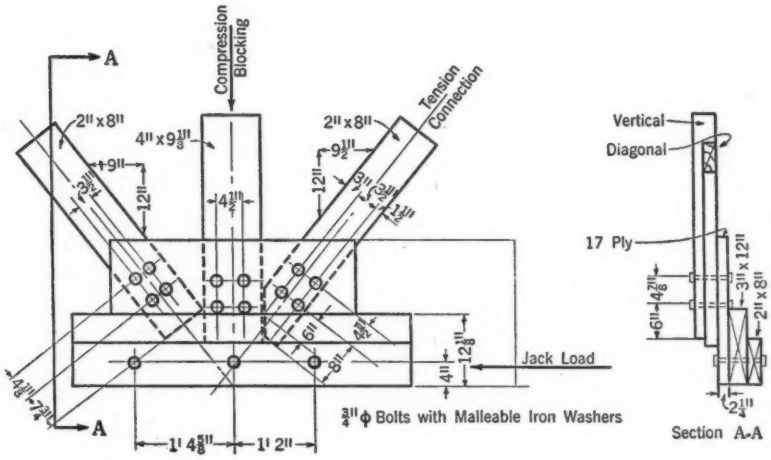


FIG. 55.—DETAILS OF JOINT J18, SAN FRANCISCO BUILDING

joint consisted of a 2¼-in.-thick seventeen-ply Douglas fir plywood gusset plate to which were connected two diagonal members, each a 2-in. by 8-in. stick, with ¾-in. bolts in single shear. (This plywood was of "exterior" type, made with phenol formaldehyde resin adhesive.) The other members of the joint (not concerned in the test) were parts of a 4-in. by 10-in. strut and the lower chords of one of the original trusses, consisting of 2-in. by 8-in. and 3-in. by 12-in. members also connected with ¾-in. bolts. The joint was in excellent condition and was tested at a moisture content, in the 2-in. by 8-in. diagonal, of about 13.5%.

The other four joints, J19, J20, J21, and J22, were constructed in the laboratory, of materials from the Exposition buildings. Their details are shown in Fig. 56. Each joint consisted of a center 6-in. by 8-in. block with two splice plates of twenty-nine-ply plywood $3\frac{3}{16}$ in. thick and 8 in. wide, connected to the 6-in. by 8-in. block with eight 4-in. split rings and four ¾-in. bolts and with a 9-in. spacing and a 7-in. end distance (see Fig. 56). The grain of the face ply ran parallel to the direction of the load in two of the tests, J19 and J20, and perpendicular in the other two, J21 and J22. The plywood was obtained from the Federal Building where it had been exposed to the weather for about 2½ years.

Bolted Joint (J18) with Plywood Gusset Plates.—Joint J18 failed in the 2-in. by 8-in. tension member by shearing out the end, the end distance being 6 in. The washers were pulled into the face of the plywood causing some splitting of the top plies and creating a corrugated effect. Meanwhile, the bolts had been bent into an S-shaped form.

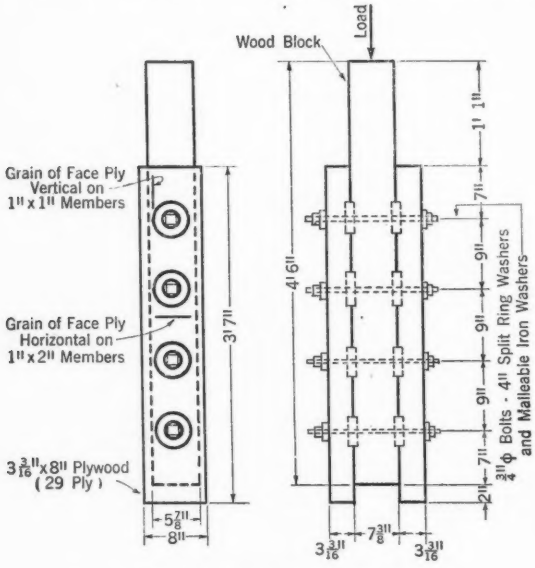


FIG. 56.—DETAILS, JOINTS J19 TO J22, INCLUSIVE

The total stress in the diagonal at failure was 42 kips. The connection of the diagonal, in which four $\frac{3}{4}$-in.-round bolts were in single shear, had a computed total safe load, based on the 2-in. by 8-in. stick, of 4.7 kips, or 1.16 kips per bolt. On the plywood at an assumed value of 45° to the grain of the wood, the four bolts have a value of 3.11 kips, or 780 lb per bolt. These values yield R-values of 8.95 and 13.5, respectively; but it is to be noticed that the slip of bolts at the ultimate load was 0.35 in. The deformation at a working load of 4.7 kips on the four bolts was 0.05 in.

The proportional limit of the bolts is estimated to have been about 8.0 kips, giving a ratio of proportional limit load to allowable, or working load, R_{bl} of 1.7, based on the bolt value of the 2-in. by 8-in. member. The corresponding slip of bolts was 0.10 in. Summarizing these results (in inches):

Based on the 2-in. by 8-in. stick—
$R_{ba} = 8.95$
$R_{bl} = 1.7$
Based on the plywood—
$R_{ba} = 13.5$
Slip of the bolts—
At allowable load $P_b = 0.05$ in.
At proportional limit load $P_l = 0.10$ in.

Compression or Tension Splice Joints with Plywood Splice Plates and Split Rings.—Joints J19 to J22, inclusive, with plywood splice plates failed in two types of failure, as illustrated in Figs. 57. Fig. 57(*a*) shows the crushing and buckling of the individual plies of the plywood when subjected to compression loads—the first type. The other type of failure is shown in Fig. 57(*b*) in which

the fibers of the plies running perpendicular to the direction of the load are seen to have sheared and "rolled" in the plywood, causing a shear failure at the base of the split rings.

After dismantling the four joints, it was found that all the ring cores in the plywood had failed and that the cores in the rings of the 6-in. by 8-in. center block remained intact. In the ring cores, about 25% of the core area failed in the glue; the remainder failed in the wood. The failures always occurred at a ply located at approximately the bottom of the ring and in a ply whose grain was perpendicular to direction of the load. The cores evidently failed by "rolling shear" action. (The term "rolling shear" has been applied

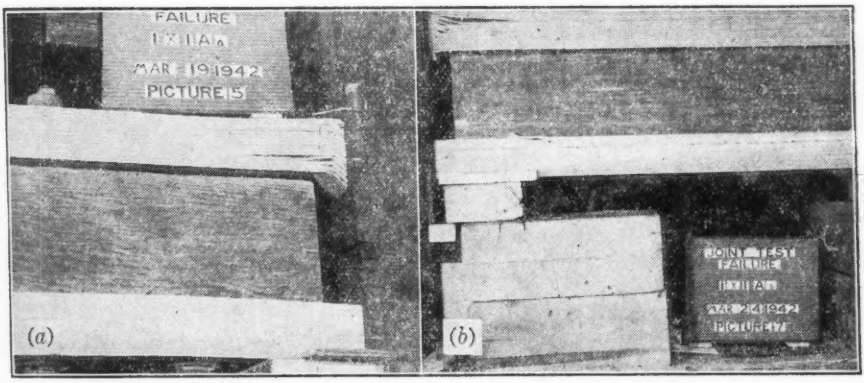


FIG. 57.—TWO TYPES OF FAILURE JOINTS WITH PLYWOOD SPLICE PLATES
(a) Crushing of Plywood
(b) Rolling Shear Failure in Lower Plywood Pad

when the shearing force tends to deform the hollow wood cells diagonally in a direction perpendicular to their fibers.)

The results of these four tests are summarized in Table 12, which shows maximum loads, manner of failure, and other pertinent data. The working values of split rings in plywood have been taken as the TECO allowable loads for group A woods with direction of load at 60° to the grain. The working capacity of plywood in compression was assumed at 1,735 lb per sq in., and the allowable rolling shear value at 60 lb per sq in.—both of these stresses being those recommended by the Douglas Fir Plywood Association based on tests by the Forest Products Laboratory.

Results of Tests.—The ratio R_{ba} of 8.95, as found in the test of joint J18, is characteristic of bolts; also a value of R_{bl} of 1.7, based on the 2-in. by 8-in. member, seems proper. The tests indicate that the joint had been proportioned correctly. The tests also indicate that an increase in the edge distances in plywood would reduce any effects of rolling shear at a critical plane, and that the plies running parallel to the loads are the effective ones in resistance to compression. It follows that the shearing and crushing strength of the plies needs particular attention in the design of joints of this type. Additional test data in connection with both bolt and connector fastenings when used

TABLE 12.—JOINTS WITH PLYWOOD SPLICE PLATES AND 4-IN. SPLIT RING CONNECTORS

Joint	Load on Joint, in Kips				R_{ca}	Deformation at allowable load (in.)	P_l	R_{cl}	Residual load[f] (kips)	Testing time (min)
	Maximum	Allowable, Based on:								
		Split rings[e]	Plywood (compression[d])	Plywood (shear[e])						
J19....	130.0[a]	40.96	45.6	41.5	3.17	0.045	61	1.49	98.8[g]	35
J20....	145.5[a]	40.96	45.6	41.5	3.5	0.048	84	2.06	83.0[h]	5
J21....	142.3[b]	40.96	42.9	41.5	3.47	0.074	82	2.00	...[h]	10.5
J22....	148.3[b]	40.96	42.9	41.5	3.62	0.041	70	1.71	73.5[h]	35[i]

[a] Applied parallel to the grain of the face plies. [b] Applied perpendicular to the grain of the face plies. [c] Based on group A wood; 60° to the grain. [d] Allowable or working loads of the joint based on plywood in compression at 1,735 lb per sq in. [e] Based on an allowable rolling shear of 60 lb per sq in. [f] After failure the joint was still capable of supporting the loads shown. [g] Failure was by buckling and crushing of the splice plate, as shown in Fig. 57(*a*). [h] Failure was by buckling and crushing of one splice plate and by rolling shear on the other. [i] 29 min one day and 6 min the next day.

with plywood, such as information on plywood splice plates and gusset plates, would be helpful in design, especially when large loads are involved.

The average deformation of joints J19 to J22, at their maximum loads at which deformations were measured, was about 0.24 in., corresponding to a load per split ring of about 30.5 kips.

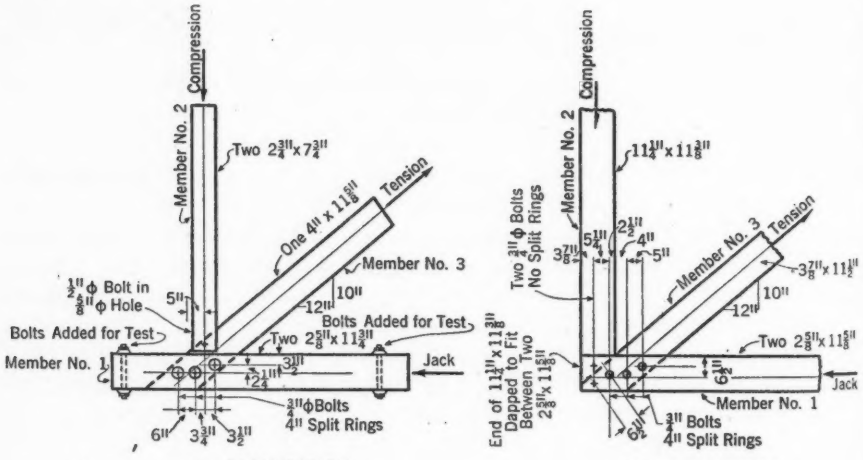


FIG. 58.—DETAILS OF JOINTS WITH SPLIT-RING CONNECTORS

Joints with Split-Ring Connections.—This series of tests comprised six joints, J23 to J28, inclusive, in which the members were connected by 4-in. split rings and bolts. Two of the joints, J23 and J24, were from an 80-ft roof truss of the Pacific Building. The other four, J25 to J28, inclusive, were compression or tension splices from the stage framing of the Cavalcade of America.

Joints J23 and J24.—One of the truss joints, J23, was taken from an interior panel of the truss, and the other, J24, formed the end joint of the same truss at its supporting column. Fig. 58 shows the details of these two joints. Diagonal member No. 3, Fig. 58(*a*), connects to chord member No. 1 by three pairs of split rings, and vertical member No. 2 was intended to bear on the top side of member No. 1.

The trusses from this building were dropped and damaged during dismantling. The following excerpt from the laboratory notes is pertinent:

> "Joint J–23 was obtained from the Pacific Building where it was an interior joint of one of the 80′ span roof trusses. It was only in fair condition for the two sticks of the chord, Member #1, were badly split. Also, Member #3 had a large check. The truss had been dropped in wrecking the building but the joint had suffered no visible damage. Because of splitting of the chord member, stitch bolts were placed as shown in Fig. 73 [58(*a*)]."

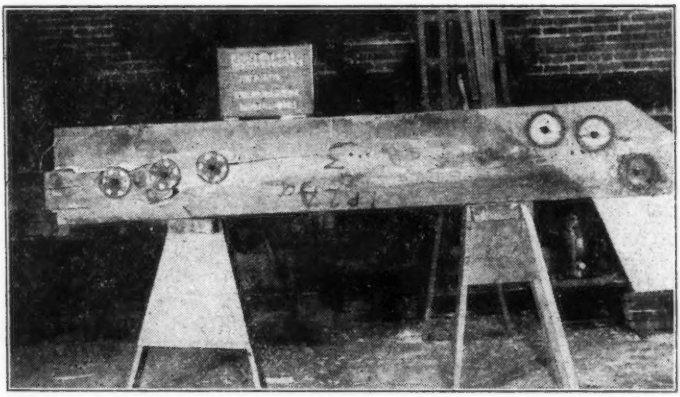


FIG. 59.—FAILURE OF JOINT J24

Although the second of these truss joints, joint J24, was in a somewhat damaged condition prior to the test, the results apparently were not affected. The members of both joints J23 and J24 were of select structural grade Douglas fir, and the two joints were tested at a moisture content of about 15%. No shrinkage was noted that would cause looseness of the bolts; but, in joint J23, there was a space of $\frac{5}{16}$ in. between vertical member No. 2 and chord member No. 1.

Joint J23 failed by splitting of diagonal member No. 3, at a total stress of 44.3 kips, or only 7.4 kips per ring, corresponding to one and one-half times the allowable load. The existing large check had opened into the wide crack. After disassembling the joint, it was noticed that all the ring cores had been sheared off before testing, except one core in member No. 1 and one core in member No. 3. These two particular cores were not even sheared at the completion of the test. The bolts were not bent. The wood in contact with the two end rings appeared to be in good condition, but the inside ring had "slid" into the check.

In the test of joint J24 (see Fig. 58(*b*)), failure did not take place in the original joint but in the tension connection installed for testing purposes at one end of diagonal member No. 3. When the connection had failed, several difficulties prevented its reconstruction, so the test was discontinued. Fig. 59 shows the member after disassembly, and the shearing out of shear plates in the tension connection may be noted. There were several small knots in this particular location. Attention is called to the grooves and cores of the split rings which remained in good condition, although the connectors had been loaded to three times their allowable loads. The maximum load on the member was 87.2 kips.

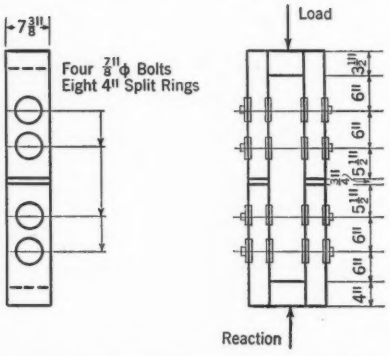


FIG. 60.—TYPICAL DETAILS OF JOINTS J25 TO J28, INCLUSIVE

Joints J25 to J28, Inclusive.—Each splice joint consisted of a 6-in. by 7½-in. center timber, spliced with two 3½-in. by 7½-in. splice pads, and it had a single row of four pairs of 4-in. split rings with ⅞-in. bolts—a total of eight split rings (see Fig. 60). The joints were in excellent condition. Shrinkage of the wood had resulted in a gap between the center stick and each splice pad of a measured amount of from $\frac{1}{16}$ in. to $\frac{5}{16}$ in.—an average of $\frac{3}{16}$ in. For convenience of testing, in which the joints were placed in compression, a ¾-in. to ⅞-in. gap was provided at the center of the length of splice pads. The movement of splice members was measured. Because of the limitation of slip of joints as determined by the ¾-in. to ⅞-in. gap between the splice pads, the maximum resistance of the split rings was not realized. Table 13 and Table 14 summarize the results, and Fig. 61 shows the stress deformation curves of the four joints.

TABLE 13.—TRUSS JOINTS WITH SPLIT RINGS

Joint	Location	Ultimate load (kips)[a]	P (kips)[b]	R_{es}	Slip (in.)[c]
J23	Intermediate Truss Joint: Tension member	44.3	28.9	1.55	....[d]
J24	End Reaction Truss Joint: Tension connection	87.23	23.8	3.65	....
	Joint	87.23[e]	28.9	3.00[e]	0.059

[a] In tension member. [b] Six split rings. [c] Slip of split rings longitudinally in member No. 3 at the design allowable load P. [d] Not evident. [e] In tension member, observed proportional limit was at a tension of 42.3 kips giving an R_{es}-value of $\frac{42.3}{28.9}=1.46$. This tension member did not fail in the joint, failure occurring in the tension connection provided for testing.

Results of Tests.—Reference to the stress deformation curves of Fig. 61, all plotted to the same scales, and also to Table 14, indicates the erratic test behavior of these joints. The slopes of the first sections of the stress-deformation curves of joints J25, J26, and J27 are very flat as compared with the

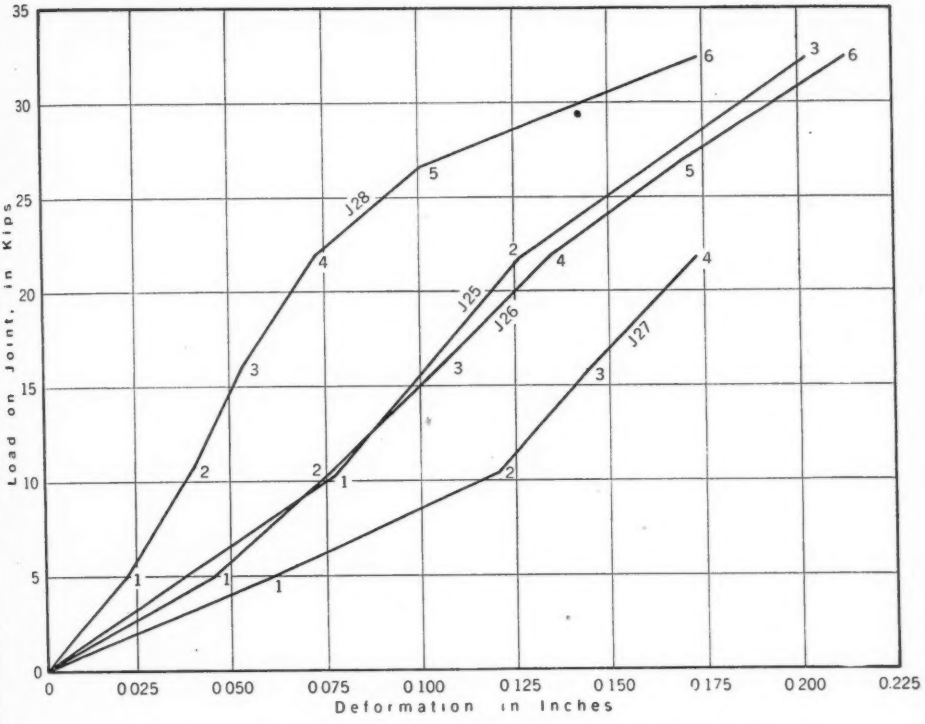


FIG. 61.—LOAD DEFORMATION CURVE, JOINTS J25 TO J28, INCLUSIVE (SPLICE WITH FOUR, 4-IN. SPLIT RINGS AND TWO ¾-IN. BOLTS ON EACH SIDE)

similar part of the curve of joint J28, indicating that a relatively large initial deformation took place in joints J25, J26, and J27. Up to a load of 10 kips the curves of joints J25 and J26 coincide, having a slope of about 4 on 7, whereas the curve of joint J27 has a relatively very flat slope. Joint J28, which

TABLE 14.—SPLICE JOINTS (J25 TO J28) WITH SPLIT RINGS

Joint	Gap between spliced members (in.)	Maximum Loads on Joint[a]		Proportional limit load[b] (kips)	Allowable load[c] P_c	Ratio[a] R_{ca}	Deformation[d]	Opening caused by shrinkage[e]
		Four rings (kips)	Kips per ring					
J25	3/16	52.5	13.1	22.0	20.2	2.6	0.119	1/8
J26	1/4	51.5	12.9	22.0	20.2	2.55	0.126	5/16
J27	7/16	51.5	12.9	....[f]	20.2	2.55	0.166	7/16
J28	1/4	55.5	13.8	22.0	20.2	2.75	0.067	1/4

[a] The joints did not fail completely, but the test was stopped when the opening between the spliced members was closed. For the maximum load per split ring there was a slip of ¼ in. [b] Values of the proportional limit load are very uncertain because of the large increments of test load. The ratio R_{ca} is equal to the proportional limit load (22.0 kips) divided by P_c, or 1.09. [c] The basic allowable load on four 4-in. split rings is 22.0 kips; but this value was first increased 12% to 24.6 kips because of the ¾-in. bolts, and then decreased to 20.2 kips (or 5.05 kips per split ring) because of the spacing of 6 in., center to center. [d] Deformation, in inches, at a load of 20.2 kips. [e] Total width of the two spaces between the splice pads and the center block, caused by shrinkage. [f] Not measured.

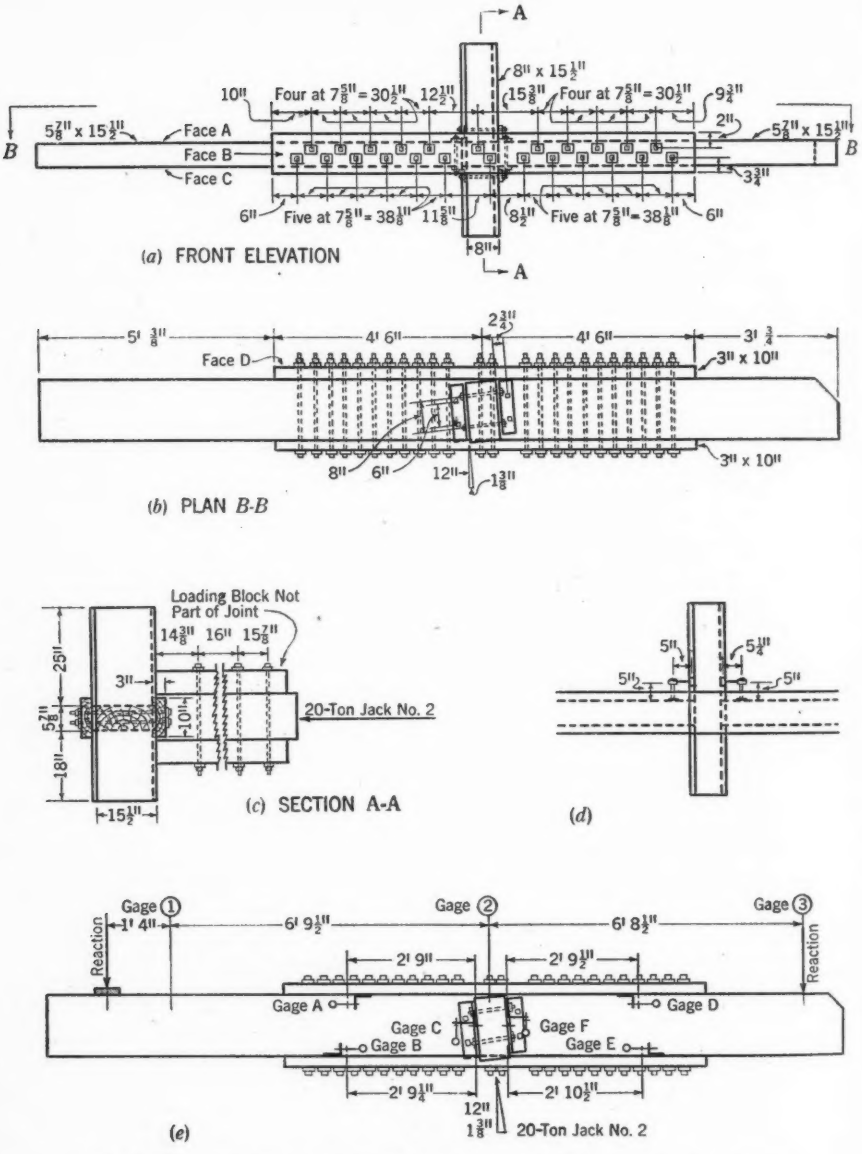
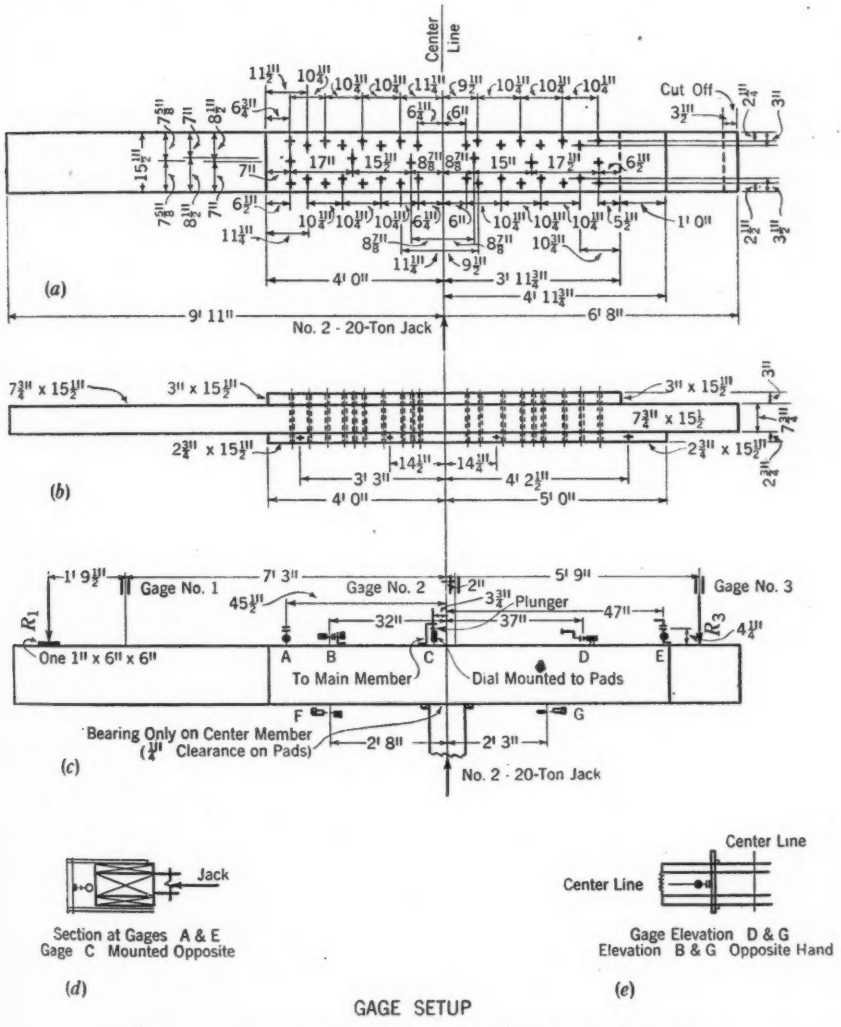


FIG. 62.—MOMENT JOINT, TYPE I (J29, J30, AND J31) (BOLTS ¾ IN.; PLATE WASHERS, ⅝ × 3 × 3; AND ANGLES, ⅝ × 4 × 4)

shows a slip of 0.06 in. at the allowable load of its connectors, $P_e = 20.2$ kips, is the only one of the four joints that exhibited normal behavior. These joints were taken apart after testing, and no defective workmanship was evident, so that the lack of consistency in their behavior cannot be explained.

Moment Joints.—The series of bolted joints J29 to J34 consisted of six splices of 6-in. by 16-in. and 8-in. by 16-in. wooden beams which had been part of the framing supporting a canvas roof over the seats of the Cavalcade. All splices had been calculated to resist moment as well as shear, and the



GAGE SETUP

FIG. 63.—MOMENT JOINT, TYPE II (J32, J33, AND J34) (STITCH BOLTS, ⅝ IN.; BOLTS, ¾ IN.; PLATE WASHERS, 5/16 × 3 × 3)

framing was designed to support a live load of 5 lb per sq ft plus dead load, with a 50% increase in the usual working stresses. Some of the beams had been continuous, with spans of 40 ft and 42 ft, whereas others had been 21-ft cantilevers extending beyond their supporting girders.

The spliced joints were of two types. Type I (joints J29 to J31, inclusive) had its splice pads on the tension and compression faces of the beams, the beam being cut in two to allow its supporting girder to pass through within its depth. It had steel angle clips bolted to its sides and to its supporting girder to resist shear. The splice pads were designed to act in direct tension or compression with their connecting bolts in single shear. Fig. 62 shows the details of this type of joint. The joints of type II, shown in Fig. 63, had their splice pads on the sides of spliced beams, the pads designed to act in bending and shear, with the bolt groups on each end of the pads furnishing resisting moment. In both types the bolts were $\frac{3}{4}$ in. in diameter with 3-in. by 3-in. by $\frac{5}{16}$-in. plate washers under the heads and nuts. These bolts were generally found to have a tight fit in their holes in the wood, but the "play" under the heads and nuts indicated a shrinkage of the wood of from $\frac{1}{8}$ in. to $\frac{1}{4}$ in.

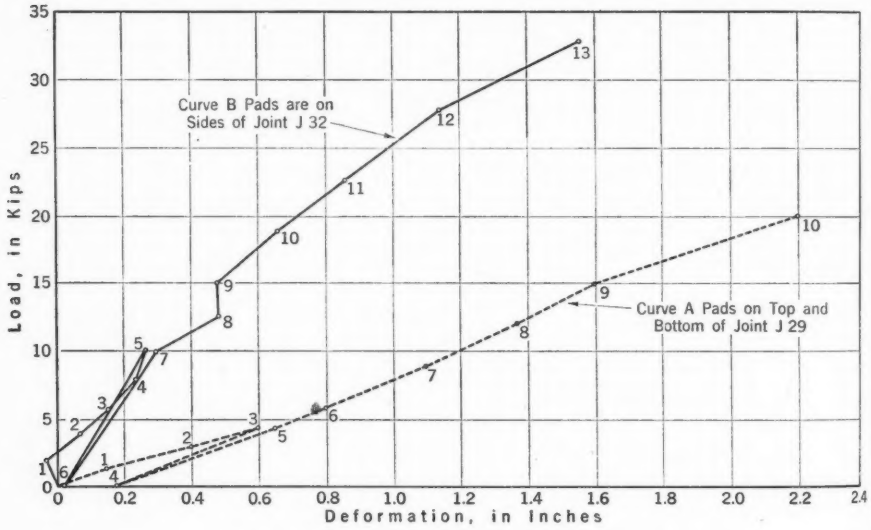


FIG. 64.—CENTER DEFLECTION OF MOMENT JOINTS

The joints had been carefully removed from the structure and were found to be in excellent condition; workmanship and materials appeared to be reasonably good, although some checking had taken place in the 16-in.-wide splice pads. The joints were tested at a moisture content of from 15% to 25%.

Testing Procedure.—After installing the joints in the testing truss, loads and reactions were applied as indicated diagrammatically in Fig. 63. In five of the joints, one concentrated load was applied directly over the splice, and in the sixth joint two equal loads were applied about 3 ft apart, near the center of the splice.

Deflections under the loads were measured on all joints; but, since these measurements were taken on one side of the beams only, some erratic results were obtained, and consequently the actual values of deflections are to be taken with caution from the deflection curves A and B of Fig. 64.

Loads were applied with an hydraulic jack and through blocking to the splice points. When loads that produced bending moments of about 20 ft-kips for type I joints and 30 ft-kips for type II joints had been applied, the loads were released, and deflections and deformations were measured. Joints were then reloaded until failure occurred.

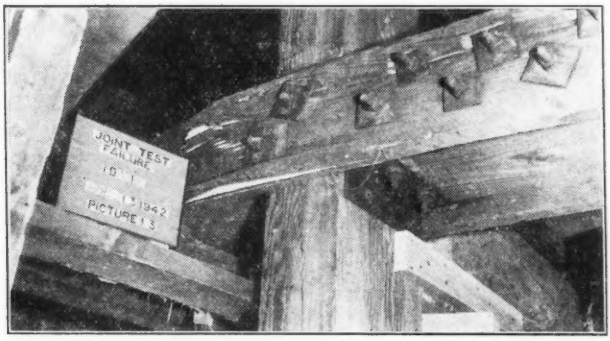


FIG. 65.—FAILURE OF JOINT J31 IN BENDING AND TENSION OPPOSITE THE SPLICE POINT

Results of Tests.—Failure in joints of type I occurred in the tension pads, caused by direct tension and secondary bending stresses. Fig. 65 shows a sharp bend in the pads which created large secondary bending stresses. No attempt was made to calculate these secondary stresses.

TABLE 15.—STRESS RATIOS, TESTS OF TOP AND BOTTOM SPLICE PADS, MOMENT JOINTS, TYPE I, J29 TO J31

Joint	Maximum supported load (kips)	Maximum moment (ft-kips)	Ultimate shear (lb per bolt)	Ratio, R_{bs}	Stress in tension pad[a] (lb per sq in.)	Estimated load P_l (lb per bolt)	Ratio, R_{bl}
J29.....	28.8	105.9	7,450	6.31	2,730	3,862[b]	3.27
J30.....	30.5	111.3	7,767	6.57	2,970	4,601	3.90
J31.....	29.8	105.3	7,471	6.31	2,740	3,515	2.98

[a] Computed stress in the tension pad neglecting bending effects. [b] Was at least 3,862 lb per bolt.

TABLE 16.—DEFORMATIONS OF TOP AND BOTTOM SPLICE PLATES, MOMENT JOINTS, TYPE 1, J29 TO J31
(For Loads and Stress Ratios, see Table 15)

Joint	Bolt Deformations, in Inches, Under the Following Loads per Bolt:					Joint Deflections[a]			Span length[b] (ft)
	984 lb	1,180 lb	3,862 lb	4,000 lb	5,394 lb	984 lb per bolt (in.)	Maximum at Center		
							(Kips)	(In.)	
J29.....	0.057	0.070	0.1670	....	....	0.52	20.1	2.21	14.75
J30.....	0.056	0.068	0.182	0.185	0.2563	0.420	30.5	3.86	14.77
J31.....	0.0477	0.060	0.164	0.1707	....	0.395	29.8	3.54	14.54

[a] At the point of application of the load. These values are to be accepted as relative only, since measurements were taken only on one side of the splice. [b] Beam section, 6 in. by 16 in.; splice pads, one each top and bottom, 3 in. by 10 in. by 9 ft; and twenty-two ¾-in. bolts.

TABLE 17.—LOADS AND DEFORMATIONS, SIDE SPLICE PADS, MOMENT JOINTS, TYPE II, J32 TO J34

Joint	Maximum load (kips)	Maximum moment (ft-kips)	Bending Stress (Lb per Sq In.)		Theoretical Bolt Stress at Ultimate Load				Horizontal Theoretical Shear in Pads (Lb per Sq In.)		Maximum Recorded Deflections[b]	
			Main member (theory)	Splice pads (computed)	If Bearing Parallel to Grain (0°)		If Bearing Across the Grain (90°)					
					(Lb)	R_{bo}	(Lb)	R_{bo}	Ultimate	First cracks[a]	(Kips)	(In.)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
J32..	27.9	101.1	4,900	5,880	13,000	5.54	5,800	4.97	1,020	330	27.9	1.249
J33..	32.8	115.3	4,460	6,010	17,300	7.3	9,000	5.92	1,040	....	27.7	1.146
J34..	33.4	96.9	4,850	5,280	12,600	5.36	4,370	2.88	443	415	26.8	1.772

[a] Computed shear at the first appearance of shear cracks. [b] To be considered relative only, because the measurements were taken on only one side of the splice.

In joints of type II, failure occurred first in horizontal shear at the pads, as evidenced by offsets at the ends. Usually this first shear failure followed existing checks, but new cracks were also opened connecting to the existing checks—in a manner quite typical of horizontal shear in beams. In a few cases the shearing cracks developed in entirely new and unchecked material. After the splice pads were split into two or more pieces by shearing, each part then broke in bending, or by subsequent splitting.

TABLE 18.—DIMENSIONS IN INCHES, MOMENT JOINTS, TYPE II (J32 TO J34), WITH SIDE SPLICE PLATES

(See Table 17)

Joint	Beam sections	Splice pads	Number of bolts ($\frac{7}{8}$-in. ϕ)
J32....	6×15$\frac{1}{4}$	2×15$\frac{1}{4}$ 3×15$\frac{1}{4}$	40
J33....	7$\frac{1}{4}$×15$\frac{1}{4}$	3×15$\frac{1}{4}$ 2$\frac{1}{4}$×15$\frac{1}{4}$	38
J34....	6×15$\frac{1}{4}$	2$\frac{1}{4}$×15$\frac{1}{4}$[a] 2$\frac{1}{4}$×15$\frac{1}{4}$[a]	32

[a] Length 14 ft.

The results of the tests are summarized in Tables 15, 16, 17, and 18, from which it may be observed that the joints with side pads (type II) are nearly two and one-half times stiffer than those of type I with top and bottom pads. Fig. 66 shows typical bolt deformation in the joints with top and bottom pads (type I). Comparing the results of Tables 15 and 16 with the bolt slips of Fig. 66, it is noted that considerable deflection of the beam took place with only small bolt slips because of the large ratio of span length to depth of beam—that is, a beam deflection of 0.4 in. corresponds to a bolt slip of 0.075 in.

In joints of type II the determination of amount and direction of shear in each bolt is a complex and indeterminate problem, unlike similar problems in steel construction, because of the difference in the elastic properties of the wood parallel and perpendicular to the grain. It may be conceived that the total moment in the splices is resisted by a combination of two types of bolt action. In the one type, assuming that the bolts act parallel to the grain of the wood only in resistance to the bending moment, each bolt would act in

double shear and the pads could probably be split along the center line of the beam with no loss in strength or rigidity. This type of action would produce tension and compression in the pads, but no horizontal shear. Col. 6, Table 17, shows the bolt stresses under such action. In the second type of bolt action,

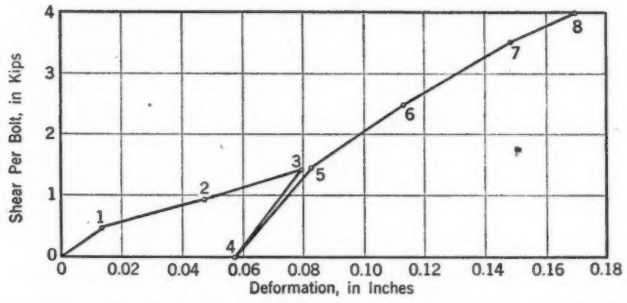


FIG. 66.—TYPICAL BOLT DEFORMATION CURVE, JOINTS OF TYPE I, WITH TOP AND BOTTOM PADS (BOLTS, ¾ IN. ROUND IN SINGLE SHEAR)

the bolts are assumed to bear perpendicular to the grain of the wood only; and, hence, their pressures would induce large shearing as well as bending stresses in the pads. Bolt stresses for this type of action are shown in Col. 8, Table 17, and the calculated horizontal shear is indicated in Col. 10.

The total resistance of such a splice must result from a combination of the two types of action. The distribution between the two types of bolt action will depend on the dimension of the joint and the relative elastic properties of the bolts and the wood in the two directions, and is highly indeterminate. Even if only a small proportion of the total bending moment is resisted by bolt action perpendicular to the grain of the wood, high shearing stresses must occur in the pads—as was evidenced in the tests of the joints of type II by the fact that all the first failures were caused by horizontal shear. Further tests and study are needed on splices of this type if a definite and rational method of design is to be determined.

The strength ratios obtained in these tests were adequate—even large. Both types of joints supported approximately the same ultimate loads, but the joints with the side pads were much stiffer. In all these tests, failure occurred in the splice pads, and not in the bolts. Probably the same loads could have been supported by using fewer bolts. In type II joints, it is probable that even larger loads could have been obtained by increasing the thickness of the pads, thus reducing the horizontal unit shearing stresses. In type I joints, secondary bending stresses, usually neglected in design, were the primary causes of failure.

Butt-Block End Joints of Trusses.—Two butt-block end, or heel, joints of identical design (J35 and J36), removed from the 80-ft span roof trusses of the Mission Trails Building, were tested to failure. They formed the connection of the lower chords of the trusses to the building columns. In each joint a heavy butt block supported the end diagonal and was connected to the lower chord with six 4-in. split rings. The members were of select structural Douglas

fir, and the joints were tested at a moisture content of 17%. A detail of one of the joints is shown in Fig. 67. Unfortunately for the test results, the butt blocks were of boxed-heart material; and, although aluminum paint had been applied to their ends, considerable checking had occurred.

Joint J35.—In the test of joint J35, when the end diagonal, member No. 1, carried a total of stress of 95 kips, the member was forced about $\frac{3}{16}$ in. into the beveled face of the butt block; and, when its stress had reached 111.5 kips, the

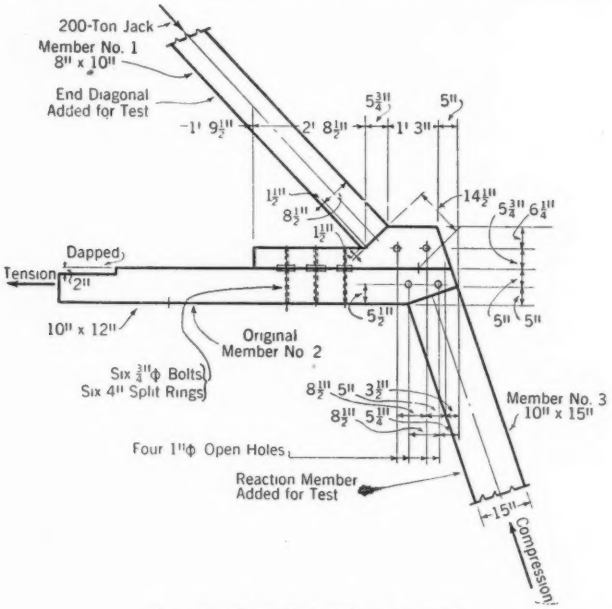


FIG. 67.—BUTT-BLOCK END JOINT J35

indentation had increased to $1\frac{5}{8}$ in. The block itself sheared and split along check planes that followed the rays of the boxed heart. The maximum slip of the split rings was 0.05 in.

Joint J36.—In the test of joint J36, to prevent failure by crushing the beveled face of the butt block, a steel plate was installed at the end of diagonal member No. 1. During the test this steel plate was pressed into the block about $\frac{1}{2}$ in., and the block was sheared on a horizontal plane about halfway up the beveled cut. In spite of these failures, the loads did not decrease appreciably, and the maximum load was recorded after the shearing occurred. This condition was undoubtedly caused by the action of the steel plate in reducing the shear on the plane of failure.

When the maximum load on the joint had been reached, it was noted that the reaction bearing of the supporting post (member No. 3), which had replaced the original column for testing purposes, had been pushed into the lower chord member from about $1\frac{1}{4}$ in. to $1\frac{1}{2}$ in. At this time no further increase in load could be obtained, but the joint continued to hold the full load for about 1 hour, at which time the test was discontinued.

TABLE 19.—BUTT-BLOCK HEEL JOINTS

Allowable, or Working Loads and Stresses:
Longitudinal shear, in pounds per square inch 150
Bearing stress at 45° with the grain, in pounds per square inch 563
Bearing stress at 70° with the grain, in pounds per square inch 380
One split ring, in pounds per split ring 5,500

Joint	Maximum Load Stresses					Reaction Bearing (70° Cut)		Ratio R_{wa} Based on		
	Shear			Beveled Face of Block (45° Cut)				Shear in the block	Bearing 45° on the block	Bearing 70° on the chord
	Six split rings[a] (kips)	One split ring[a] (kips)	Unit shear at base of block (lb per sq in.)	Total load (kips)	Stress (lb per sq in.)	Total load (kips)	Stress (lb per sq in.)			
J35...	75.8[b]	12.6	277	111.5	1,640	86.0	574	1.85	2.93[e]	1.51[e f]
J36...	96.2[b]	16.0	296[c]	139.5[d]	1,453[d]	105.7	729	1.97	2.57[e]	1.91[e]

[a] Neglecting friction effects. [b] Split rings did not fail. [c] The base block sheared at this load but the load did not decrease. [d] Steel plate had been added. [e] These factors are of doubtful value since the working stress in bearing, on inclined planes, should be based on the indentation in the wood. [f] Did not fail.

Results of Test.—The results of the tests are summarized in Table 19 in which the various maximum and working loads and stresses, corresponding unit stresses, and ratios R are shown. The working stresses of 563 lb per sq in. and 380 lb per sq in. (Table 19) are based on the formula developed by R. L. Hankinson:[6]

$$N = \frac{P\,Q}{P \sin^2 \theta + Q^2 \cos^2 \theta} \qquad (1)$$

in which P is the allowable unit compressive stress parallel to the grain; Q is the allowable unit compressive stress perpendicular to the grain; N is the allowable unit compressive stress at angle θ to the grain; and θ is the angle to the grain of applied load.

Joint J35 failed by shear in the butt blocks after the diagonal had been impressed about $1\frac{5}{8}$ in. into the 45° cut of the block. The split rings were intact whereas the block was completely split.

As in joint J35, the block in joint J36 was split, and failed in shear; but the load did not decrease. The reaction bearing (the post) sank $1\frac{1}{4}$ in. into the 70° bevel cut of the chord. Despite the steel plate, the 45° bevel dug an impression $\frac{1}{2}$ in. deep.

If the working load for which the end diagonal was designed is taken as the inclined component of the working capacity of the six 4-in. split rings, which fastened the butt block to the chord, such axial and diagonal stress would have been 46.5 kips. The corresponding unit stress on the beveled face of the butt block would have been 580 lb per sq in.; and the corresponding unit shearing stress in the block, 150 lb per sq in. Judged by the failure of the butt block, these working stresses were too high for the quality of butt block used. A bearing stress of 450 lb per sq in. and a shearing stress of 90 lb per sq in. would

[6] "Investigation of Crushing Strength of Spruce at Varying Angles of Grain," *Circular No. 259*, U. S. Air Service Information, Washington, D.C., 1921.

have been more in order. These values would have given *R*-values of about 3, based on the ultimate shearing stress and on the butt-block bearing stress at which an indentation of $\frac{3}{16}$ in. was observed. A boxed-heart stick should not have been used for so important a member as the end butt block, for such a stick is liable to serious checking.

Even had the block been of sound material, failure at such comparatively low unit stresses leads one to question whether the generally accepted working stresses for longitudinal shear in joints and bearing on inclined surfaces are not too high. Probably the detail of block could have been modified to provide a lower shearing stress; but, to reduce the bearing stress, a larger end post would have been necessary.

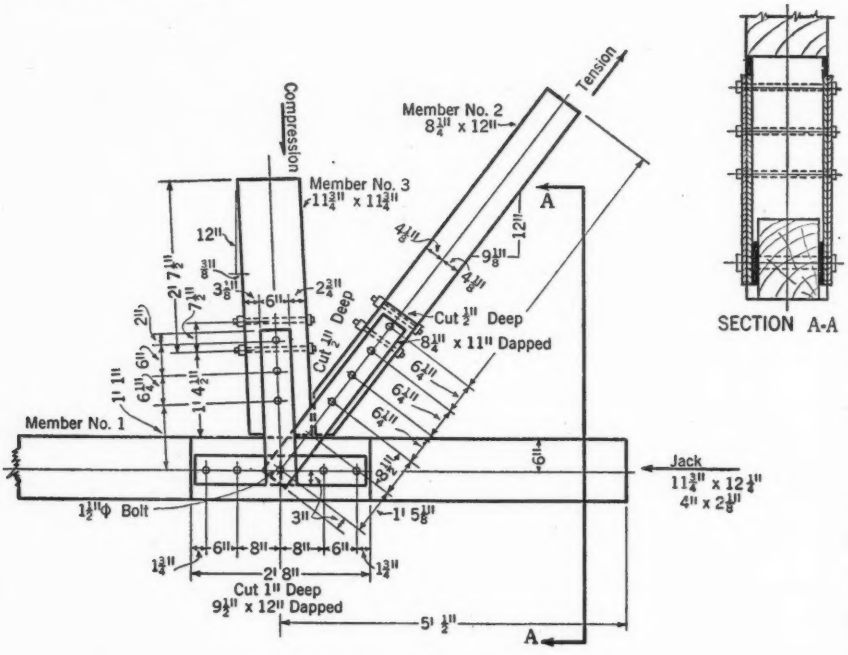


FIG. 68.—STRAP AND PIN JOINTS—JOINT J37

Strap and Pin Joints.—Tests were made of three joints of similar design (J37, J38, and J39) in each of which a pair of steel pin plates were bolted to each member entering the joint; and each pair of pin plates was then connected to a single pin. One of the joints is shown in Figs. 68 and 69. The pin plates were 6 in. by $\frac{3}{8}$ in. Each pair was fastened to each side of each wood member with 1-in. round bolts, and interconnected with a single $1\frac{1}{2}$-in.-round pin. The joints were in excellent condition; and the wood was of select grade Douglas fir and was tested at a moisture content of about 15%. Joints J37 and J38 had some checks at the bolts in their diagonal tension member—member No. 3. Shrinkage was not apparent in joint J37; but shrinkage in joint J38 was estimated at about $\frac{3}{8}$ in., whereas that in joint J39 was estimated at $\frac{1}{8}$ in. All

bolts were tight in their holes in the wood. The joints were provided with ½-in. stitch bolts as shown in Fig. 68.

The results of the tests are summarized in Table 20. Fig. 70 shows the slip of steel plates with respect to the wood in tension member No. 3, joint J39,

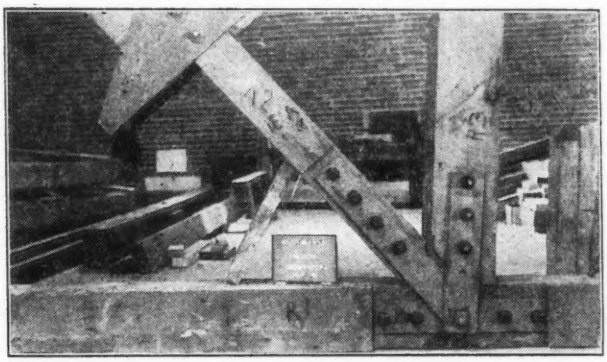


FIG. 69.—STRAP AND PIN JOINT BEFORE TESTING

which is the member that failed. Fig. 71(*a*) shows the typical splitting of this tension member at the ultimate load, and Fig. 71(*b*) shows the same failure after the steel plates had been removed. From Table 20, it can be noted that

TABLE 20.—BOLTED STRAP AND PIN JOINTS

Joint	Loads on Members, in Kips						Ratio,[a] R_{bs}	Dap of Bolts[b] (in Inches)		End distance[c] (in.)	Residual load (kips)	Elapsed time[d] (min)
	Maximum, at Failure			Allowable, P_b				At P_b	At end of test			
	No. 1	No. 2	No. 3	No. 1	No. 2	No. 3						
J37..	40.2	50.6	63.7	21.0	15.7	21.0	3.03	0.068	3½	8½	61.8	41
J38..	50.2	46.5	69.3	21.0	15.7	21.0	3.31	0.070	1½	9½	29.6	23
J39..	95.7	84.1	123.4[e]	21.0	15.7	21.0	5.9	0.012	0¼	9¼	71.0	128

[a] Ratio R_{bs} for member No. 3, which failed. [b] Measured on one side of joint only. [c] End distance on the first bolt at member No. 3. [d] Elapsed time of testing. [e] The tension connection used consisted of six 4-in. shear plates with ¾-in. bolts, 12-in. end distance, and 6-in. spacing. In joint J39, this connection successfully withstood 123.4 kips, the largest maximum load of the three tests. P for six shear plates equaled 30.5 kips without reducing for substandard spacing and 23.2 kips if reduced for 6-in. spacing. Thus, $R_{bs} = \frac{123.4}{23.2} = 5.32$.

the R-values of the failed members were 3.03 in joint J37, 3.31 in joint J38, and 5 and 5.9 in joint J39. The values of the proportional limit loads were not determined. Joint J39 took a greatly increased ultimate load as compared to the loads taken by joints J37 and J38.

As stated, there were checks in the tension members of these last two joints which may have lowered the ultimate loads; but it scarcely seems possible that these slight difficulties were responsible for the great difference in load-carrying capacity. After failure, in spite of the splitting shown in the photographs, the joints continued to carry considerable load, as shown in Table 20.

After dismantling, it was found that the bolts nearest the pins were bent the least, whereas the bolts farthest from the pins had larger deformations, possibly due to the action of the stitch bolts. During the tests the members began splitting at the ends; therefore, it is believed that, if the stitch bolts had been

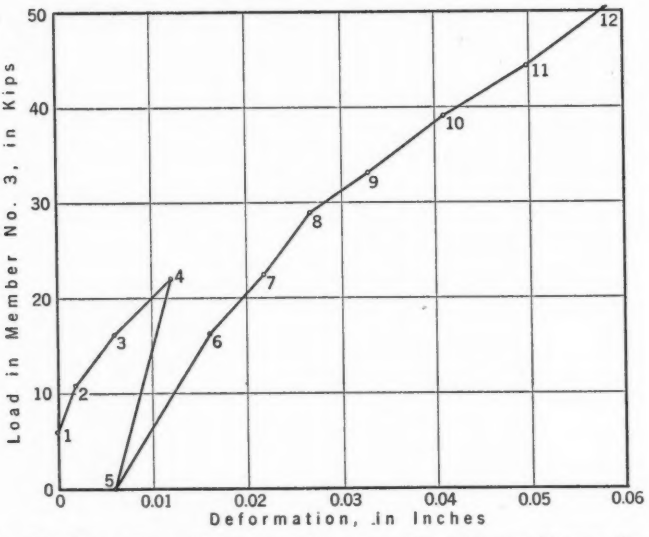


FIG. 70.—SLIP OF STEEL PLATES WITH RESPECT TO WOOD TENSION MEMBER NO. 3

placed at the ends of the members, their ultimate loads would have been greater. The excess of the ultimate loads of joints J37 and J38 is unexplainable, except perhaps that it furnishes an example of the old adage that "the exception proves

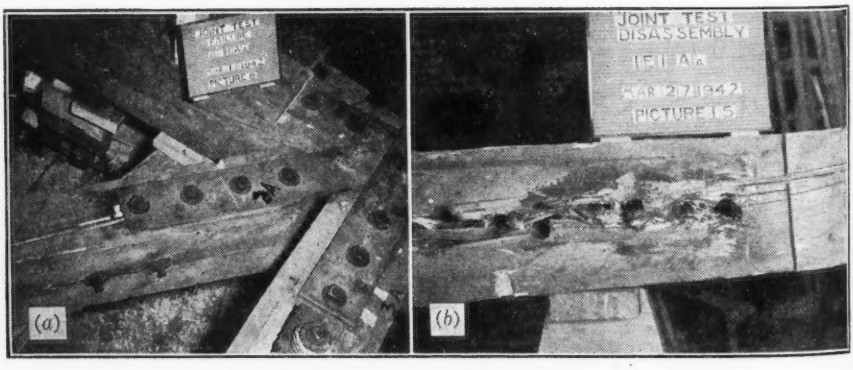


FIG. 71.—TEST OF TENSION MEMBER NO. 3
(*a*) View of Split in Tension Member at Failure
(*b*) Same as (*a*) with Steel Plates Removed

the rule." The increased capacity is apparently not the result of a better quality of wood. Member No. 3 of joint J39 had a boxed heart; it had 25% summerwood as against from 30% to 50% summerwood, respectively, in the

same members of joints J37 and J38; and it had twenty rings to the inch as compared to fourteen and thirty eight, respectively, in joints J37 and J38.

Bolted Joints with Steel Straps.—Tests were made of two joints of identical design taken from the East Cambodian Towers in which the members were interconnected by steel straps and bolts as shown in Fig. 72.

Diagonal member No. 3 consisted of two 3-in. by 14-in. sticks with an 8-in. by 14-in. filler block between. The member was connected to two steel straps, each $\frac{1}{2}$ in. by 12 in., with three $1\frac{1}{8}$-in. bolts. The horizontal member (member No. 1) was a 14-in. by 16-in. stick with four $1\frac{1}{8}$-in. bolts connecting to the same steel straps, the bolts bearing at an angle of $34\frac{1}{2}°$ to the grain of the member.

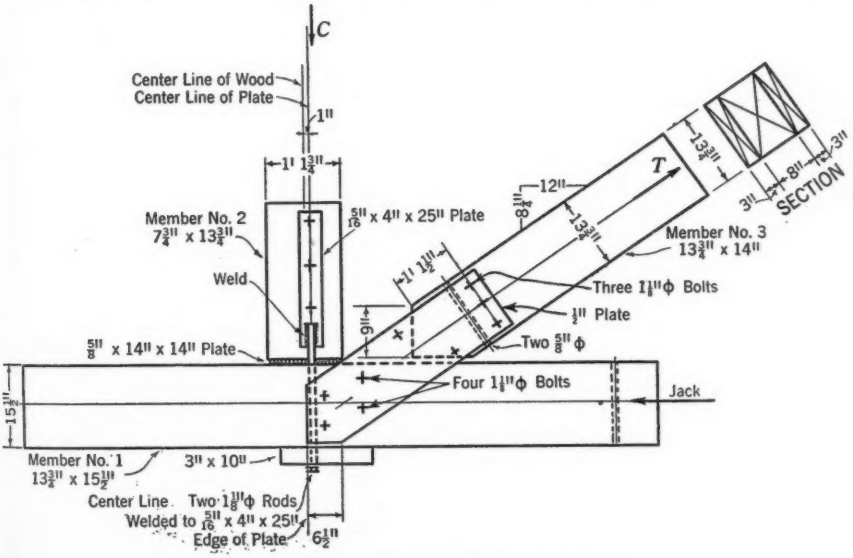


FIG. 72.—BOLTED JOINT WITH STEEL STRAPS

Vertical member No. 2, an 8-in. by 14-in. stick, bore on a steel plate on the top of member No. 1 and in addition had a steel tension connection to the same member. Joint J40 is detailed in Fig. 72. The wood of both joints was of select structural grade of Douglas fir in good condition. The measured shrinkage amounted to about $\frac{1}{16}$ in. in 14 in. of width. At the time of testing the wood had a moisture content of about 15%.

In the test of joint J40, the tension connection was made to the two 3-in. by 14-in. diagonals using the 8-in. by 14-in. block between them as a filler. In the test of joint J41, the pull was applied directly to the 8-in. by 14-in. stick, so that the two 3-in. by 14-in. sticks acted as loose fills in the transfer of loads to the steel plates. In joint J40, failure took place not in the original joint, but in the new tension connection of member No. 3.

In the test of joint J41, loads were applied until the deformation became so large that the joint merely yielded without further increase in load. Consequently, the test was discontinued because of these large deformations, with the joint still holding its maximum load. The joint was then taken apart and

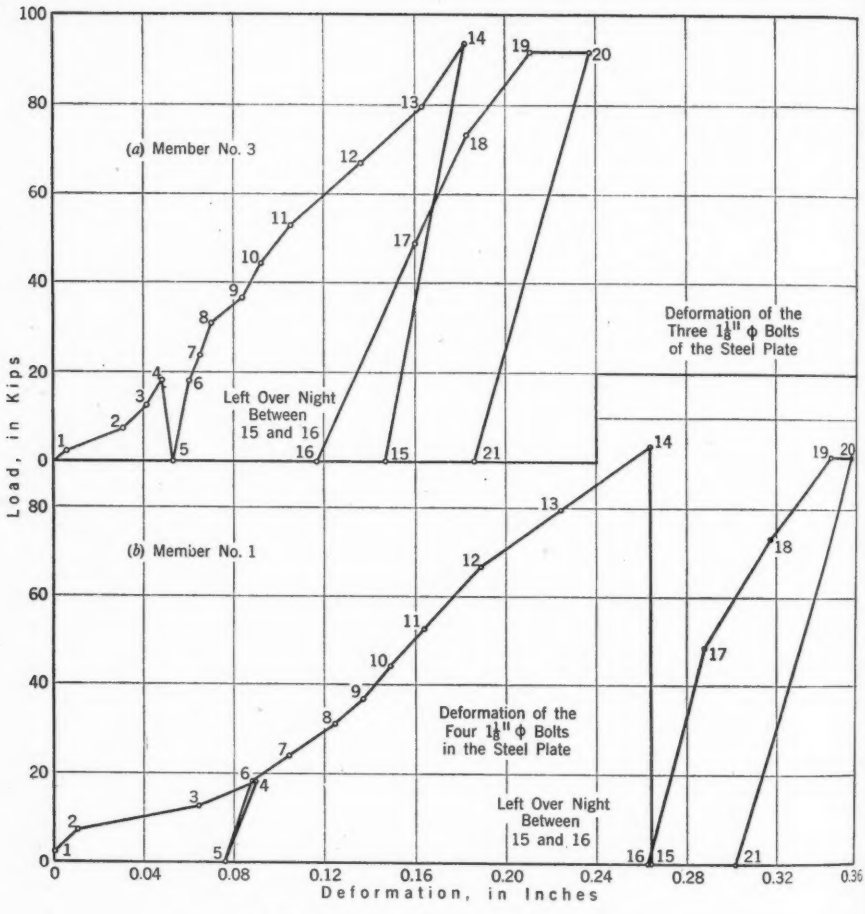


FIG. 73.—STRESS-DEFORMATION CURVE OF BOLTS

TABLE 21.—BOLT AND STEEL-STRAP JOINTS

Joint	Maximum stress[a] member No. 3 (kips)	Allowable Loads P_b on Total Number of 1⅛-In. Bolts		Stress Ratios			Residual load, member No. 3 (kips)	Load P_i in member No. 3	Bolt Deformation (Inches)				Elapsed time (days)
				R_{ba}		R_{ca}			At Load P_b		End of Test		
		Member No. 3[b]	Member No. 1[c]	Member No. 1	Member No. 3	Testing connection			Member No. 1	Member No. 3	Member No. 1	Member No. 3	
J40..	93.9	19.9	18.4	5.1	4.7	4.3	85.4	66.7[b,d]	0.089	0.048	¼	⅜	2
J41..	129.2	19.9[e]	18.4	7.0	6.5	....	129.2	...[d]	0.051	0.050[f]	⅝	1¼	2

[a] The tension member. [b] Three bolts, loads parallel to the grain. [c] Four bolts, loads at an angle of 34½° to the grain. [d] Not evident. [e] Through two loose filler plates. [f] Includes loose filler plates.

the blocks split open to observe the deformations of the bolts, which amounted to $1\frac{1}{2}$ in. at points of maximum bending. Fig. 73 shows the stress deformation curves of the joints and Table 21 summarizes the results. The slips of bolts in both joints at their allowable loads were the normal slips (about 0.05 in.), but the influence of the loose fillers in joint J41 is evidenced in the much larger slips of the bolts in this joint at their maximum load.

III. Column Tests

Built-Up Columns.—

Spaced Column and T-Section Columns.—Because of financial limitations, the somewhat extensive original program of tests of built-up columns and struts had to be curtailed so that only three built-up columns were tested; however, they represented three separate types.

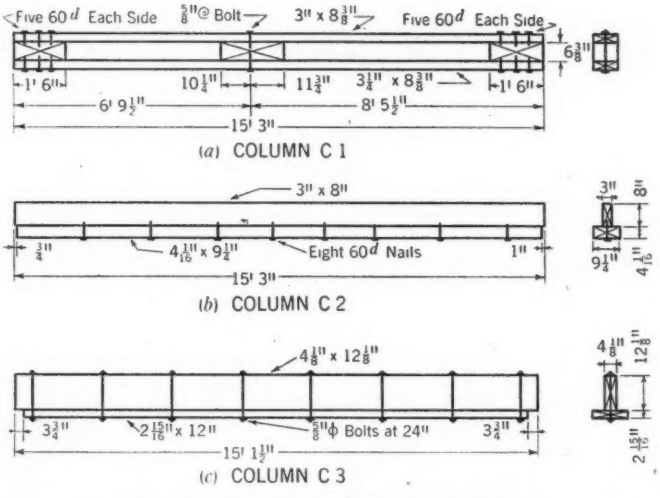


Fig. 74.—Details of Built-Up Timber Columns

Column C1, taken from the Mission Trails Building, was made of two 3-in. by 8-in. sticks spaced 6 in. apart, held together at midheight by a block with a single $\frac{5}{8}$-in. bolt and at the ends by end blocks fastened only by nails. Column C2 was from the San Francisco Building, and column C3 was from the Mission Trails Building, where it had formed the top chord of the bracing truss for the main framing of this building. Each of the two columns consisted of two sticks forming T-sections, the two pieces of column C2 held together by nails and those of column C3, by bolts. Details of the three columns are shown in Fig. 74. The columns, about 15 ft long, were of select structural grade of Douglas fir and were tested at a moisture content of about 15%. The shrinkage of column C1 was estimated to be about $\frac{3}{8}$ in., and that of column C3 was from about $\frac{1}{8}$ in. to $\frac{1}{4}$ in.

The ends of all columns were carefully squared. In columns C2 and C3 of the T-section, the flanges of the T-section were cut back about 2 in. so that the

loads would bear on the vertical web only. All three columns were installed horizontally in the testing machine and the loads applied through steel plates bearing on the ends, as spherical or rolling bearing blocks were not available. Gages were installed to measure deflections caused by buckling, as well as compression deformations. Loads were applied in increments of from 15 kips to 20 kips, about 20 min being consumed for testing each column.

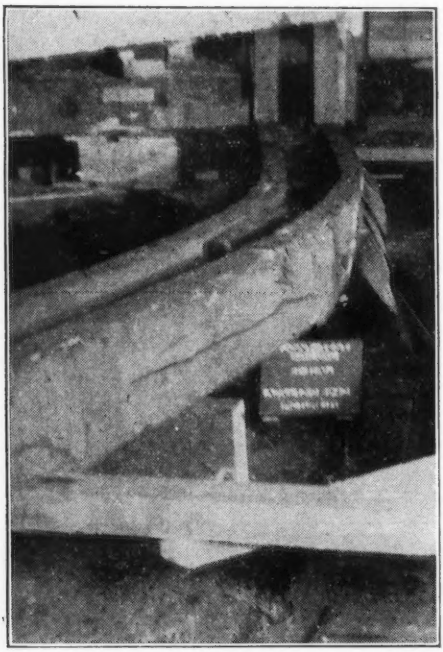


FIG. 75.—BOWING FAILURE OF COLUMN C1

Column C1, the spaced column, started buckling at a load of about 50 kips, but supported a maximum load of 60 kips. Then the resistance to load decreased, and the buckling increased. The manner of failure is illustrated in Fig. 75.

In testing column C2, of the nailed T-section, at a load of 70 kips, there was a visual indication of a sine curve with about $\frac{1}{16}$-in. amplitude. At a load of 106.3 kips, the web of the T-section suddenly buckled and tore loose from the flange, the nails ripping chunks of wood out of the web section.

Column C3, with a bolted T-section, did not fail in buckling. At a load of 182.7 kips, the head of the jack which bore directly on the web of the column crushed the wood. Just before failure, it was noted that the free edge of the web was buckling very slightly.

Results of Test.—The results of tests are summarized in Table 22. Fig. 76 shows the relationship between the center buckling and the supported load for column C1. The column had a definite proportional limit load of 50 kips. At this load the computed value of E was 1,600,000 lb per sq in. The individual members of this column had an L/d-ratio of 61 corresponding to an L/r of 210 (L = height of column; d = least depth of section; and r = least radius of gyration). The maximum unit stress at failure was 1,146 lb per sq in.

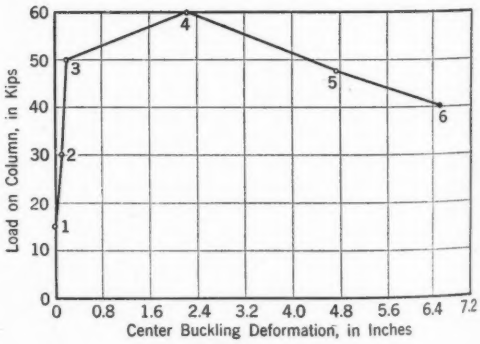


FIG. 76.—LOAD-BUCKLING CURVE OF COLUMN C1

TABLE 22.—TESTS OF BUILT-UP COLUMNS (MOISTURE CONTENT, 14%, AND PARALLEL GRAIN)

Column	Specific gravity	Rings per inch	Percentage summerwood	Length (ft)	Buckling Began at:		Maximum Supported Load		Ratios L/d		Allowable load (lb per sq in.)[a]	Ratio R_{wa}
					Load (kips)	(Lb per sq in.)	Load (kips)	(Lb per sq in.)	Web only	T-section		
C1	0.46	14.5	25	15.25	50.0	955	60.0	1,146	..[b]	...[b]	295[c]	1,146 ÷ 295 = 3.9[d]
C2[e]	0.55	20	28	15.25	...[f]	...[f]	106.3	4,431[g]	61	19.7	1,178[g]	4,431 ÷ 1,178 = 3.75
C3[e]	0.49	28	42	15.12	...[h]	...[h]	182.7	4,677[g]	44	15.1	1,193[g]	4,677 ÷ 1,193 = 3.92

[a] Modulus of elasticity $E = 1{,}600{,}000$ lb per sq in.; allowable $s = 1{,}200$ lb per sq in; and $L/d = 61$. [b] For the entire length of column C1, $L/d = 61$, the maximum between the block and the end being 30.7. [c] For a spaced column, using split-ring connectors. [d] $R_{wa} = 955/295 = 3.3$. [e] Loads were applied only to the web (vertical) of the T-section. [f] Buckling began at failure, web tearing loose from the flange. [g] Unit stress in the web section. [h] This specimen did not fail as a column and did not buckle. It failed by local crushing under the jack, with a bearing pressure of 6,172 lb per sq in.

The nailing of column C2 proved adequate to support the web, and the bolting of column C3 was even more effective. These three tests are too few in number to warrant drawing any definite conclusions. However, the loads that they carried are impressive, particularly when it is considered that: (1) They had seen continuous service for about two and one-half years; (2) they were fastened only with nails and bolts; (3) the shrinkage of the wood had rendered the bolts ineffective in tension; and (4) the bolts were not tightened for the tests. The tests tend to lend confidence in these types of columns, particularly for use as bracing struts, for which they are usually employed.

The characteristics of the wood of these columns as shown in Table 22 are interesting.

IV. PLYWOOD TESTS

This division of the test program comprised the investigation of plywood sheathing when used as a structural element resisting lateral forces, like winds and earthquakes. Plywood-sheathed walls were so utilized in a number of the Exposition buildings.

When the test program was first organized, it had been expected to test large plywood panels from the various buildings, but this procedure was later found to be impracticable. As an alternative, it was decided to investigate three basic requirements in the design of plywood diaphragms. Therefore, test specimens were built from plywood panels that had been exposed to the elements at the Exposition site for a period of about 2 years, and three groups of tests were made as follows:

Group 1.—Strength of nail fastenings under lateral loading.

Group 2.—On the shearing strength of plywood.

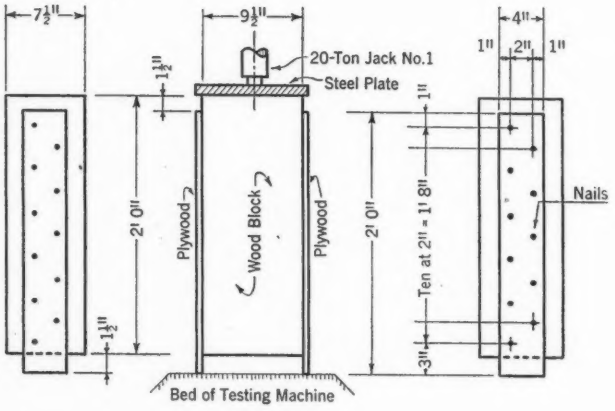
Group 3.—On the buckling of the plywood under shear loads with various spacings of studs.

All three groups of tests were run on all three of the following types of plywood:

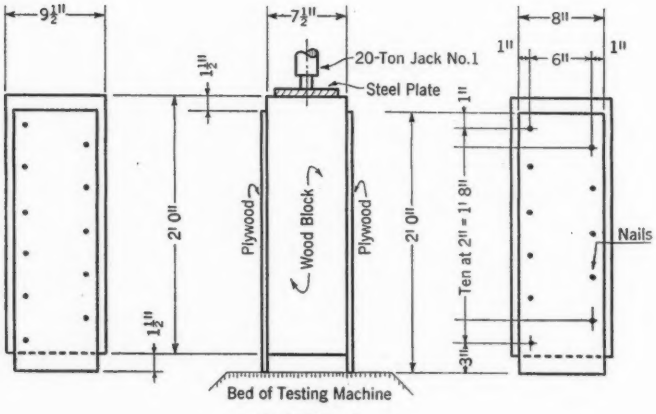
Type 1.—Three-ply $\frac{3}{8}$-in.-thick Douglas fir plywood unpainted panels that had been used in the interior of buildings and were in fair condition. Glue was a moisture resistant adhesive of the soya bean or casein type.

Type 2.—Three-ply $\frac{7}{16}$-in.-thick Douglas fir plywood specimens removed from exterior walls. They had one coat of paint and were generally in good condition.

Type 3.—Five-ply $\frac{7}{16}$-in.-thick Douglas fir plywood specimens removed from exterior walls. They had one coat of paint and were generally in good condition.



(*a*) TYPE A



(*b*) TYPE B

FIG. 77.—DETAILS OF PLYWOOD NAIL SPECIMENS

Glues for types 2 and 3 were phenolic resin adhesives or phenol-formaldehyde synthetic resin adhesives.

The preceding determinations were made in the Tacoma laboratory of the Douglas Fir Plywood Association, from samples submitted by the committee.

Group One; Lateral Strength of Nails in Plywood.—An important criterion of the strength of the average plywood diaphragm is the lateral strength of the nail fastenings to the studs. Before designing the Exposition structures, preliminary tests had been made which indicated that six-penny nails used in plywood $\frac{7}{16}$ in. thick failed first by bending and then by the heads pulling through the plywood. Consequently, when a design called for heavy shears (up to about 1,000 lb per lin ft) to be carried on plywood diaphragms, special roofing nails were specified, with a heavy No. 10 gage shank $1\frac{3}{4}$ in. long, to resist in bending and bearing, and a large head, $\frac{7}{16}$ in. in diameter, to resist pulling through the plywood. For heavy loads, these nails proved to be economical since their number was reduced. These roofing nails, together with more usual types, were tested in this series.

Specimens.—Eighty-two specimens were made up as shown in Fig. 77, including the three types of plywood mentioned previously. Some were to be tested with the loads parallel, and some with the loads perpendicular to the grain of the face plies. Nails were four-penny, six-penny, and eight-penny common wire nails, and two types of special roofing nails—all galvanized. One roofing nail was $1\frac{3}{4}$ in. long, No. 10 gage, with $\frac{7}{16}$-in.-round head, and the other was identical except that its head was $\frac{5}{16}$ in. in diameter. Nails were driven "home." On specimens of type B that had less than twenty-two nails, the top nails shown in Fig. 77(*b*) were omitted. There was no change in the number of bottom nails. The interior block which received the point of the nail was cut from an 8-in. by 10-in. piece that had a moisture content of 14% and a specific gravity of 0.476. All center blocks for the specimens were cut from the same 8-in. by 10-in. piece, so that little variation in results should be expected from this source. Special care was taken in cutting and assembling the specimens to have square cuts and even bearings for distributing the loads equally among the nails. Each specimen had from 14 to 22 nails, so that each test represents the average action of several nails.

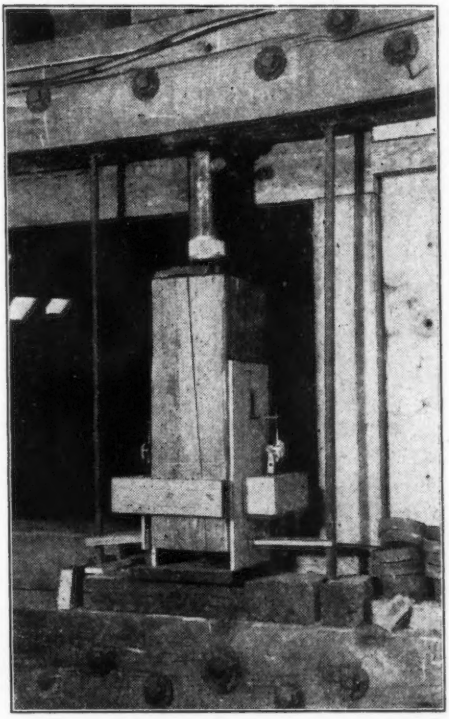


FIG. 78.—VIEW OF PLYWOOD NAIL TEST SPECIMEN IN THE TESTING FRAME

Testing.—Specimens were tested in a special part of the large testing frame, using a 20-ton hydraulic jack for applying the load. A view of a specimen in

the testing frame is shown in Fig. 78. Two Federal dial gages were used, with the wood mountings as shown in the photograph. These gages were attached to the specimen to measure the deformation between the center block and each piece of plywood.

Because hand pumping was used with the hydraulic apparatus, the speed of testing was indeterminate. Pumping was done at an approximately uniform rate. It required from about 2 min to 2½ min to reach the limit of the dial gages—indicating a deformation of about 0.25 in. Dials were then removed and about 2 min more were required to reach a total deformation of from ½ in. to ⅞ in., at which point the tests were stopped.

Type A specimen (see Fig. 77) was used for all the first tests. Trouble was experienced with some of these specimens, in that the plywood failed in column action, before the nails failed. To remedy this condition, type B specimens were used because of their increased compression area, the number of nails being decreased in some cases. Some specimens with heavier nails were braced, as shown in Fig. 78, to prevent a side movement if the plywood tended to buckle.

Failures observed in these tests were of three types. The majority of the failures—most of those for the six-penny, eight-penny, and the special roofing nails with $\frac{5}{16}$-in. heads—failed because the nails bent and then the head pulled through the plywood. This also happened in the case of the four-penny nails with $\frac{3}{8}$-in. plywood. In the remainder of the four-penny nail tests, on the $\frac{7}{16}$-in. plywood, failure generally occurred because the nail bent and then pulled out of the block to which the plywood was nailed. The third type of failure occurred because the nails bent and then the heads broke and pulled off. This result happened in all the tests of the special roofing nails with the $\frac{7}{16}$-in. heads.

At a total deformation of from ½ in. to ⅞ in., all the nails had passed the point of maximum resistance and were yielding while supporting less than their previous maximum loads.

Results of Tests.—Fig. 79 contains the load-deformation curves of the tests and Table 23 is a summary of the results. The individual tests of each group show a definite uniformity in the great majority of cases. In ten of the fifteen groups, the values for the loads acting at 90° to the grain of the face plies are larger than those acting parallel. This condition is most noticeable for light nails and for the three-ply plywoods. In the case of the five-ply plywood and larger nails, these values are closer together.

The proportional limits of the nail strength values, as given in Table 23, are very approximate since nail tests invariably yield irregular curves. In establishing safe working nail values for practical uses, it appears desirable to base them on an allowable deformation, such as 0.01 in.

Further economies in plywood construction might be realized if nails with heavier shafts and larger heads were to be manufactured. Economies in labor as well as in materials would also result since large nail spacings could be used.

Group Two; Plywood Shear Tests.—The shear resisting ability of the plywood in plywood diaphragms is of importance; yet few data on this subject are available. For this reason panels were made and tested in which the plywood had been nailed to wood studs in a most substantial manner for the purpose of

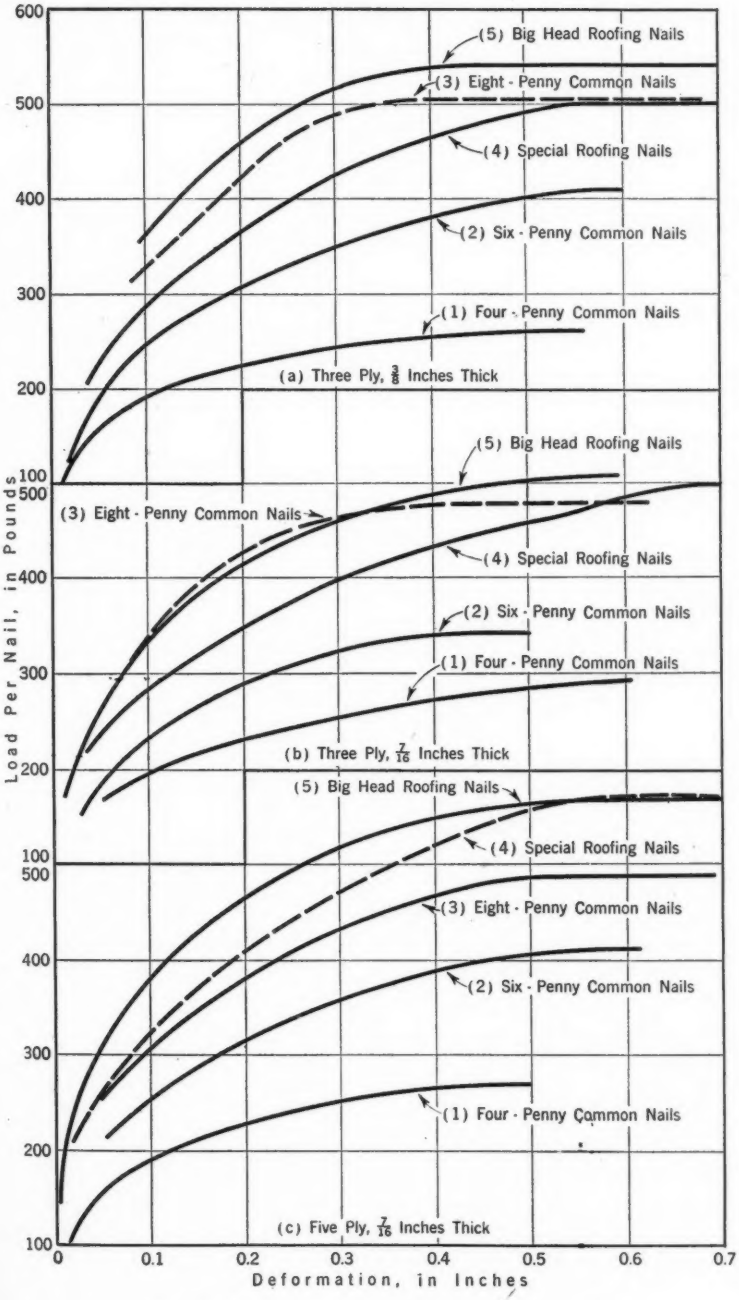


FIG. 79.—LATERAL STRENGTH OF NAILS IN PLYWOOD

TABLE 23.—OBSERVED LOADS, IN POUNDS, SUMMARY OF NAIL TESTS ON PLYWOOD SPECIMENS

Description	Three Ply, $\frac{3}{8}$ In.		Three Ply, $\frac{7}{16}$ In.		Five Ply, $\frac{7}{16}$ In.		Average value	Load Based on Factor of:			
	Parallel to the grain	Cross grain	Parallel to the grain	Cross grain	Parallel to the grain	Cross grain		1.6 on P_1	In 0.01 slip	1.33 on P_1	In 0.02 slip
(*a*) Four-Penny Nails											
Ultimate load	234	305	278	326	258	291	282	...	...	...	...
Proportional limit, P_1	105	113	105	105	92	123	108	68	...	81	...
Observed Load at Deformation of:											
0.01 in.	105	117	96	113	90	122	108	...	68	...	...
0.02 in.	125	144	119	135	112	145	130	...	...	...	81
(*b*) Six-Penny Nails											
Ultimate load	363	424	374	317	450	385	385	...	...	...	...
Proportional limit, P_1	110	155	123	137	147	148	137	86	...	103	...
Observed Load at Deformation of:											
0.01 in.	110	140	108	143	157	138	133	...	83	...	...
0.02 in.	130	172	132	174	184	171	160	...	...	...	100
(*c*) Eight-Penny Nails											
Ultimate load	520	489	474	495	501	484	492	...	...	...	...
Proportional limit, P_1	195	203	157	209	203	185	191	120	...	144	...
Observed Load at Deformation of:											
0.01 in.	172	194	154	197	187	175	179	...	112	...	...
0.02 in.	210	228	190	232	220	205	213	...	...	...	135
(*d*) Special Roofing Nails with $\frac{7}{16}$-In.-Round Heads											
Ultimate load	440	500	486	469	554	582	505	...	...	...	...
Proportional limit, P_1	167	151	172	155	187	202	172	107	...	129	...
Observed Load at Deformation of:											
0.01 in.	149	145	166	127	188	174	158	...	99	...	...
0.02 in.	185	175	201	167	216	213	193	...	...	...	119
(*e*) Special Roofing Nails with $\frac{7}{16}$-In.-Round Heads											
Ultimate load	520	565	494	525	561	506	541	...	...	...	...
Proportional limit, P_1	220	220	180	189	250	212	212	133	...	159	...
Observed Load at Deformation of:											
0.01 in.	205	194	163	214	190	195	193	...	121	...	...
0.02 in.	249	240	219	251	248	258	244	...	...	...	145

preventing either failure by buckling of the plywood sheets or failure in the nails.

Specimens.—There were three test specimens of plywood panels—a typical one is shown in Fig. 80. Exterior load-carrying members were made of wide timbers, permitting sufficient roofing nails to be applied at each edge so that the nails themselves would not fail. The panels were also blocked to prevent the plywood from buckling. Fig. 81(*a*) shows the top. At the two corners, where loads were not applied, a gap was left so that the frame itself would not carry any load. Loads were applied directly to the frame on two of the diagonal corners. To prevent bearing of the loads on the plywood itself, edges

were clipped on a diagonal thereby providing bearing for plates on the frame only.

The general setup is shown in Fig. 81(*b*). The hydraulic jack is shown at the left, and the reaction into the testing truss at the right is taken through a

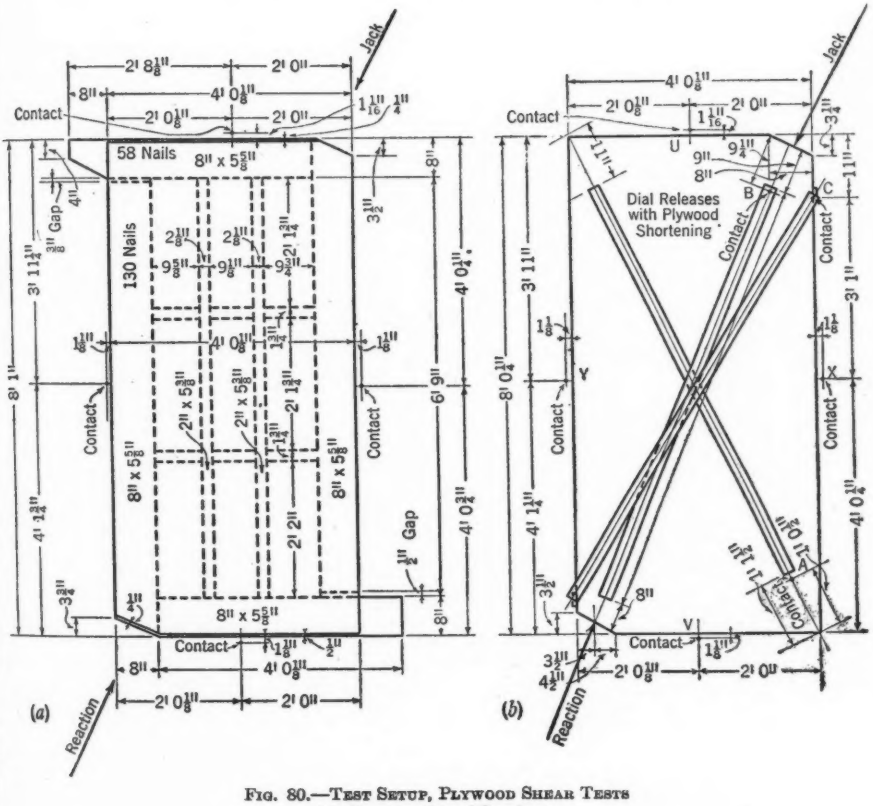


FIG. 80.—TEST SETUP, PLYWOOD SHEAR TESTS
(*a*) Construction of Specimen
(*b*) Location of Gages

short heavy beam. Gages were installed as indicated in Fig. 80(*b*). One dial gage A measured the lengthening of the tension diagonal, and another gage B measured the shortening of the entire frame from timber to timber as close to the corners as was practicable. Four gages were installed to measure the nail slip. As these slips were small and correspond to the series of tests on nails, the results are not discussed. Fig. 81(*b*) shows the dial gage setups as seen from the back of the machine. Testing procedures for each panel were as follows:

Test No. 1; Three-Ply Panel, $\frac{3}{8}$ In. Thick.—This panel was placed in the machine and loaded with a 50-ton hydraulic jack. In the first 21 min of testing, the deformation at gage C, measuring the total frame and panel deformation, was 0.331 in. At this point, with 19,800 lb on the panel, the dial gages were removed and the loading continued until a deformation of about 1 in. was

obtained after an interval of about 5 min. At this time the panel failed in shear at a load of 23,000 lb, corresponding to a shear of 2,571 lb per ft, or 575 lb per sq in. At failure, the panel split lengthwise in many fine lines. After failing, the panel still held a load of about 19,800 lb.

Test No. 2; Three-Ply Panel, $\frac{7}{16}$ In. Thick.—This panel was loaded in the same manner as in test No. 1. In the first 20 min of loading, gage C registered a total deformation of 0.241 in. Failure was reached in 6 more min with a total deformation of 0.380 in. at a load of 37,500 lb, corresponding to a shear of 4,068 lb per lin ft, or 775 lb per sq in. At this point, the panel completely "tore"

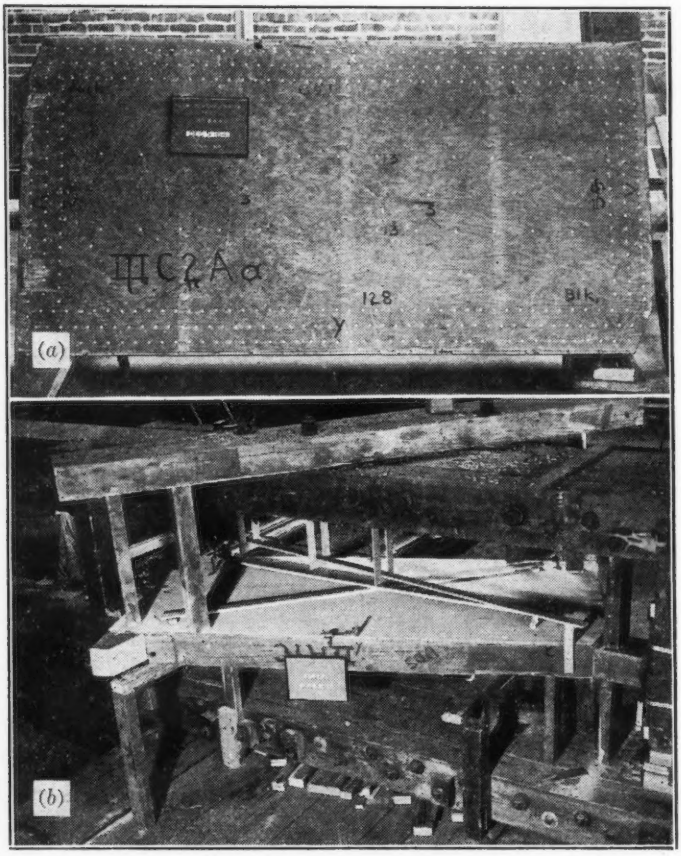


FIG. 81.—PLYWOOD SHEAR PANEL
(*a*) View of Top of Panel
(*b*) General Test Setup

at the third points. Loading was continued until a deformation of 2 in. was noted, the load dropping gradually to about 20,000 lb, which was maintained until the end of the test. After the load was removed, the panel looked as shown in Fig. 82, with offsets of about $\frac{3}{4}$ in.

Test No. 3; Five-Ply Panel, $\frac{7}{16}$ In. Thick.—The panel was installed in the testing frame and testing was started with a 20-ton jack. Loading was begun and 24 min were required to reach a maximum load of 32,170 lb, with a deformation of 0.239 in. at gage C and a corresponding shear of 3,597 lb per lin ft. Since it was found impossible to increase the foregoing load because of external conditions, it was released, noting a permanent deformation of 0.043 in. at gage C. No detrimental effects were noted. The following day, testing

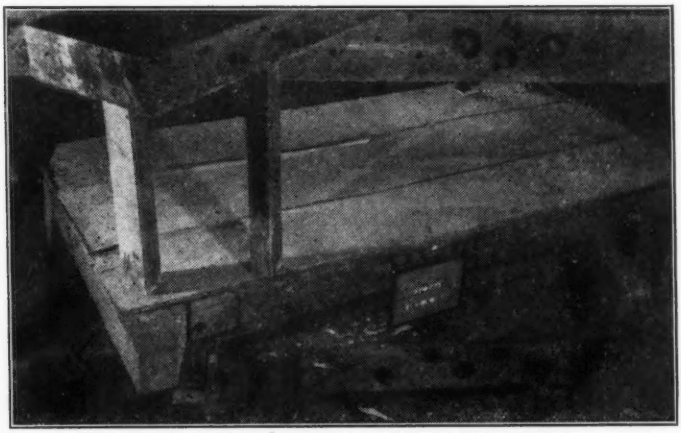


FIG. 82.—SHEAR FAILURE OF THREE-PLY PANEL, $\frac{7}{16}$ IN. THICK

continued with a 50-ton jack which required 20 min to show a total deformation of 0.278 in. at a load at failure of 51,300 lb—corresponding to a shear of 5,736 lb per lin ft or 1,090 lb per sq in. Failure consisted of a single tear down the middle of the panel.

Results of Tests.—For all three panels, Fig. 83 shows curves relative to the lengthening and shortening of the diagonals under load in terms of a 12-in. gage length and shears per linear foot acting on the plywood. The horizontal displacement of the wall (Δ) caused by plywood deformation can be computed with the aid of these curves and the formula

$$\Delta = 2.5\,K\,L \qquad (2)$$

in which (see Fig. 83(*b*)) L is the height of the panel; and K is a factor derived from the curves (deformation at design shear). All the times given in Fig. 83(*c*) are for deformation on the compression diagonal. It may be noted that the tension diagonal deformed slightly more than the compression diagonal, which seems to agree with other tests on timber and plywood. Deformations, caused by shear in the plywood, are relatively small, especially compared to nail slips. In most designs they may be neglected.

Table 24 presents a summary of ultimate loads and deformations. According to these tests, five-ply plywood, as compared with three-ply plywood of the same thickness, has about 40% greater ultimate shearing strength. The results of these tests would indicate that the capacity of plywood in shear has

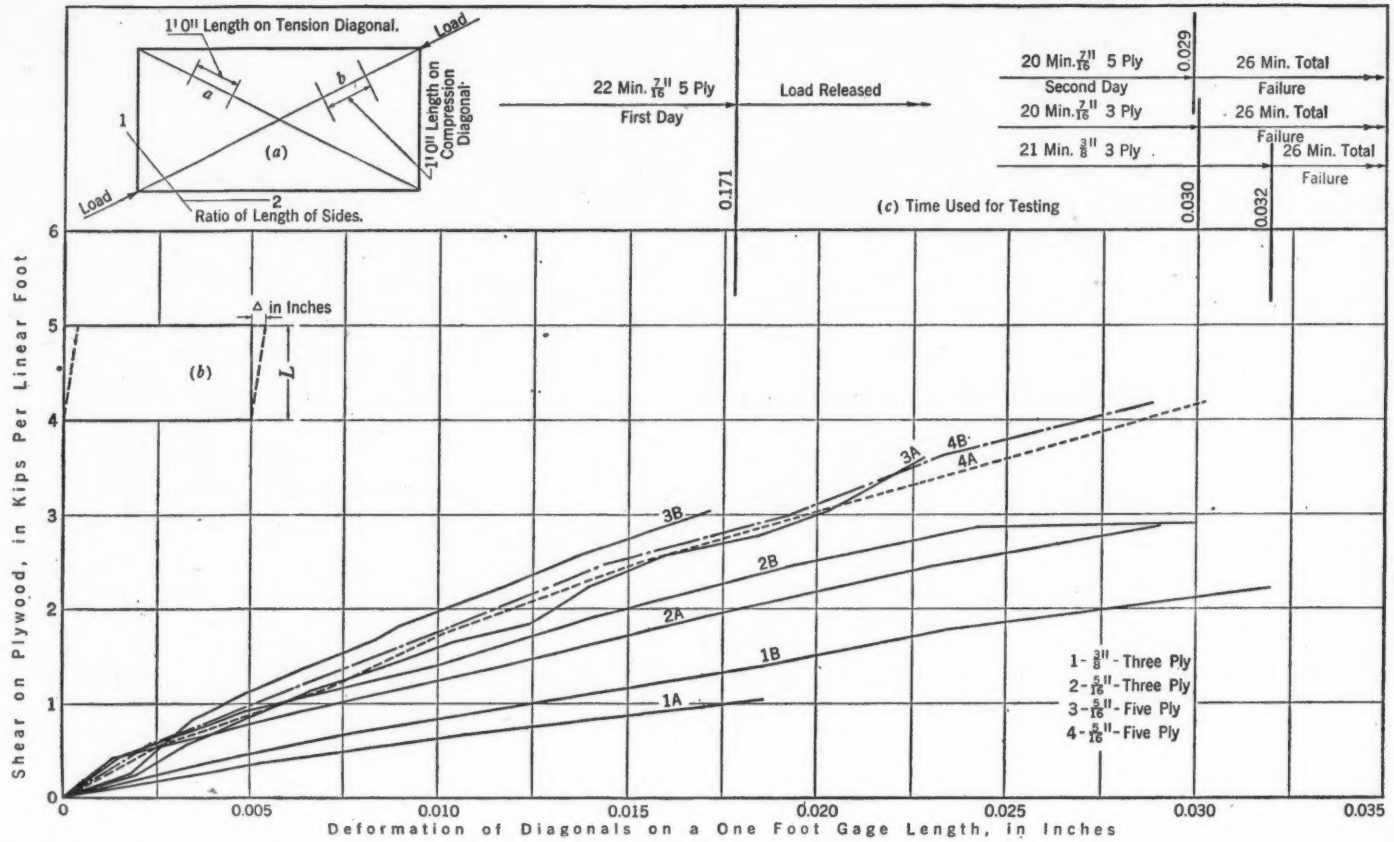


FIG. 83.—SHEAR-DEFORMATION CURVES PLYWOOD SPECIMENS, GROUP TWO TESTS

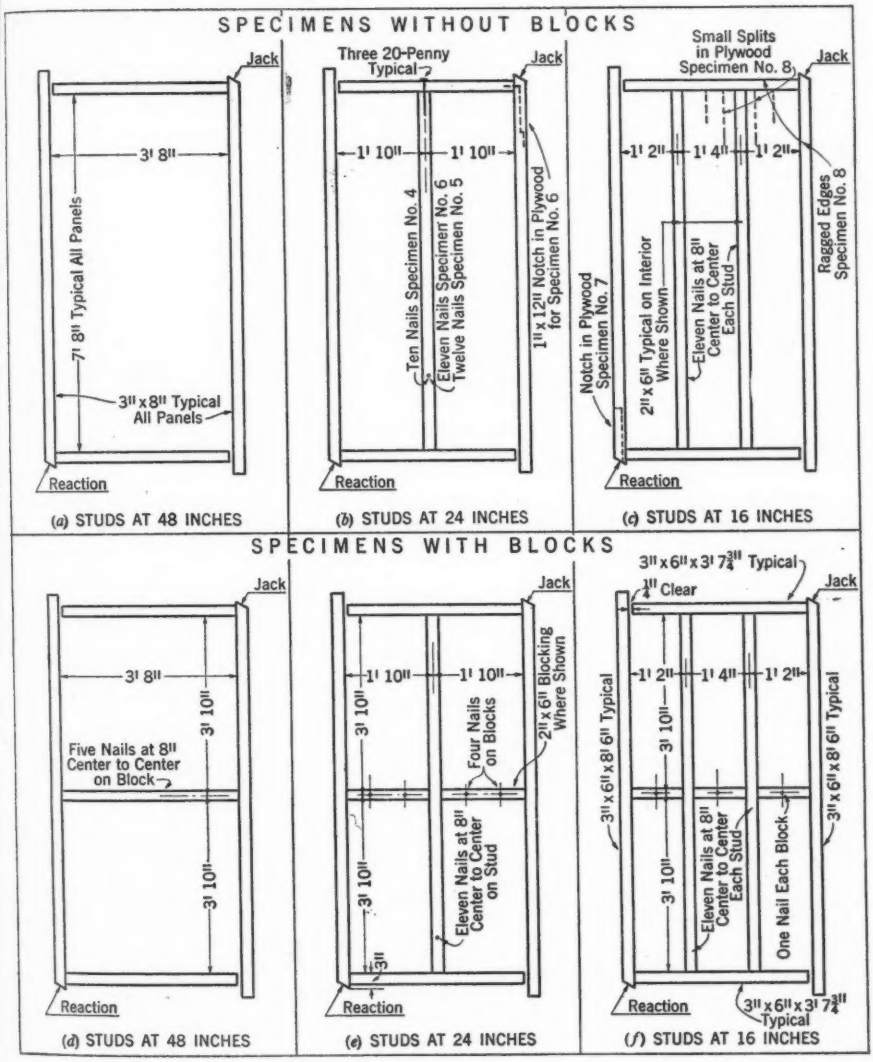

SPECIMENS WITHOUT BLOCKS
Jack
3' 8"
7' 8" Typical All Panels
3" x 8" Typical All Panels
Reaction
(a) STUDS AT 48 INCHES
Three 20-Penny Typical
1' 10"
1' 10"
Ten Nails Specimen No. 4
Eleven Nails Specimen No. 6
Twelve Nails Specimen No. 5
1" x 12" Notch in Plywood for Specimen No. 6
(b) STUDS AT 24 INCHES
Small Splits in Plywood Specimen No. 8
1' 2"
1' 4"
1' 2"
Notch in Plywood Specimen No. 7
2" x 6" Typical on Interior Where Shown
Eleven Nails at 8" Center to Center Each Stud
Ragged Edges Specimen No. 8
(c) STUDS AT 16 INCHES
SPECIMENS WITH BLOCKS
Five Nails at 8" Center to Center on Block
(d) STUDS AT 48 INCHES
Four Nails on Blocks
2" x 6" Blocking Where Shown
Eleven Nails at 8" Center to Center on Stud
(e) STUDS AT 24 INCHES
3" x 6" x 3' 7 3/4" Typical
1/4" Clear
3" x 6" x 8' 6" Typical
One Nail Each Block
(f) STUDS AT 16 INCHES

FIG. 83.—SHEAR-DEFORMATION CURVES PLYWOOD SPECIMENS, GROUP TWO TESTS

FIG. 84.—DETAILS OF PANELS TESTED FOR BUCKLING (SEE TABLE 25)

not been fully utilized because of the difficulty of making adequate connections. This deficiency may be remedied by the use of specially manufactured nails, as previously mentioned, by gluing, or by some other equally efficient method. Because of the increase in the use of plywood, these problems appear to be worthy of further study so that its potential strength values can be utilized more economically.

Group Three; Plywood Buckling Tests.—This group of tests included the investigation of plywood panels in which the buckling of the plywood itself

TABLE 24.—SUMMARY OF RESULTS, PLYWOOD SHEAR TESTS, GROUP TWO

Plywood	Load on panel (lb)	Shear		Total Deformation,[a] Gage C (in Inches)		
		(Lb per lin ft)	(Lb per sq in.)	1,000 lb	2,000 lb	Ultimate load
Three ply, $\frac{3}{8}$ in. thick	23,000	2,571	575	0.085	0.28	0.469
Three ply, $\frac{7}{16}$ in. thick[b]	37,500	4,068	775	0.053	0.123	0.380
Five ply, $\frac{7}{16}$ in. thick	51,300	5,736	1,090	0.019	0.106	0.278

[a] Total deformation (in inches) of the plywood and frame at gage C, Fig. 80, under three loads: A shear of 1,000 lb per lin ft; a shear of 2,000 lb per lin ft; and, at an instant before reaching the ultimate load. [b] See Fig. 82.

might be the strength criterion. For this purpose, eighteen specimens were built up of plywood $\frac{3}{8}$ in. and $\frac{7}{16}$ in. thick, using stud spacings of 16 in., 24 in., and 48 in., with and without blocking at the midheight of the panels. One test was made of each type of specimen.

TABLE 25.—REPORT OF BUCKLING TESTS IN PLYWOOD PANELS

No.	Plywood (see Fig. 84)	Number of Nails on Each Side (Fig. 85(*b*))				Ultimate load[a] (lb)	Shear (lb per lin ft)	Nail Value[b]
		X	Y	U	V			Pounds per nail at ultimate
	(*a*) Specimens Without Blocks							
1	Stud Spacing, 48 In. (Fig. 84(*a*)):							
	Three ply, $\frac{3}{8}$ in. thick	32	32	15	15	14,940[a,c]	1,670	445
2	Three ply, $\frac{7}{16}$ in. thick	32	32	15	15	15,420[a,d]	1,724	460
3	Five ply, $\frac{7}{16}$ in. thick	32	32	14	14	14,940	1,670	477
	Stud Spacing, 24 In. (Fig. 84(*b*)):							
4	Three ply, $\frac{3}{8}$ in. thick	32	32	14	14	15,920[a,d]	1,780	509
5	Three ply, $\frac{7}{16}$ in. thick	32	32	14	14	20,640	2,308	659
6	Five ply, $\frac{7}{16}$ in. thick	32	32	14	14	18,580	2,077	594
	Stud Spacing, 16 In. (Fig. 84(*c*)):							
7	Three ply, $\frac{3}{8}$ in. thick	31	31	14	14	15,920	1,780	509
8	Three ply, $\frac{7}{16}$ in. thick[e]	32	32	14	14	15,920	1,780	509
9	Five ply, $\frac{7}{16}$ in. thick	32	32	14	14	20,640	2,308	659
	(*b*) Specimens with Blocking							
	Stud Spacing, 48 In. (Fig. 84(*d*)):							
10	Three ply, $\frac{3}{8}$ in. thick	31	31	14	14	19,100[a,f]	2,135	617
11	Three ply, $\frac{7}{16}$ in. thick	32	32	14	14	17,700[a,g]	1,979	565
12	Five ply, $\frac{7}{16}$ in. thick	32	32	14	14	19,220	2,149	614
	Stud Spacing, 24 In. (Fig. 84(*e*)):							
13	Three ply, $\frac{3}{8}$ in. thick	32	32	15	15	13,500[h]	1,509	402
14	Three ply, $\frac{7}{16}$ in. thick	32	32	14	14	15,920	1,780	509
15	Five ply, $\frac{7}{16}$ in. thick	32	32	15	15	14,000[i]	1,565	417
	Stud Spacing, 16 In. (Fig. 84(*f*)):							
16	Three ply, $\frac{3}{8}$ in. thick	33	33	15	14	20,100	2,247	620
17	Three ply, $\frac{7}{16}$ in. thick	32	32	14	14	16,430	1,839	525
18	Five ply, $\frac{7}{16}$ in. thick	32	32	15	14	19,600	2,192	605

[a] In only five cases (Nos. 1, 2, 4, 10, and 11) was there any visible tendency to buckle. The nails failed in each case (except as noted by footnotes [b] to [g]). [b] Ultimate nail value (pounds per nail). In specimens Nos. 1, 4, 10, and 11 the nails were not the cause of failure (see footnote [e]). [c] Failure by buckling only. [d] The specimen buckled first and then the nails failed. [e] Ragged edges and ragged spots on top and left sides (see Fig. 84(*c*)). [f] The buckling test was stopped when the load on the specimen would increase no further. [g] Failure was by buckling but the nails were well bent and pulling through. [h] Failure was by nails pulling through the plywood. [i] Test was stopped when the clipped edges touched each other. [j] Nails did not fail completely.

Specimens.—The eighteen panels, measuring 4 ft by 8 ft, were constructed as shown in Table 25 with Figs. 84 and 85. Greater than usual edge distances of the nails were used because all plywood sheets already had a series of nail holes at the edges. In addition, some of the edges were slightly ragged. As in the shear tests, a space was left between framing members to minimize the amount of load that the frame itself would carry. Edges of the plywood were clipped so that no part of the jack or reaction forces would bear directly on the

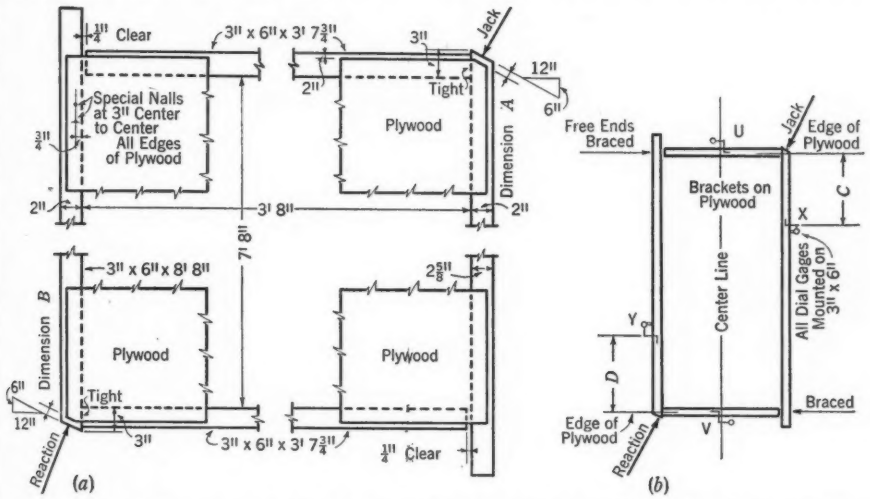


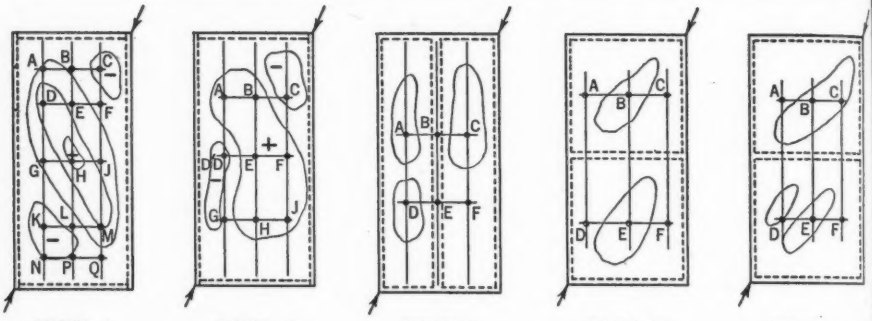
FIG. 85.—CORNER DETAILS AND KEY TO TYPICAL DIMENSION IN TABLE 25, FOR ALL BUCKLING TESTS

plywood, routing forces thereby through the frame and the nails. The plywood was fastened to the frame with special roofing nails (No. 10 gage $1\frac{3}{4}$ in. long, with $\frac{5}{16}$-in. heads) spaced 3 in. center to center at the edges and 8 in. center to to center on interior contacts. All nails were driven "home."

Testing.—The panels installed in the testing frame were like those used in the shear tests. Loading was applied with a 20-ton hydraulic jack, in a manner similar to that described for the shear tests (Fig. 81(*b*)). Gages were placed at the four edges of the plywood to measure the nail slips. Braces were required to hold the frame in position. Since the web is off center from the chords, the near corner of the panel (Fig. 81(*b*)) tends to move up whereas the corners under the jack and reactions tend to move down. Loads were applied in increments, interrupted long enough to take readings of the dial gages. Panels were tested to failure, or until the deformations were large enough that the edges of the plywood bore on the bearing blocks at the reaction of the jack.

Thirteen of the eighteen panels failed by yielding of the nails (Table 25), most of them pulling through the plywood. In a few cases, the nails pulled out of the studs. In these thirteen tests, several panels failed after holding loads at large deformations for some time. Final failure sometimes occurred from a secondary effect, the block nearest the jack rotating and splitting at large nail deformations. In five specimens, failure is attributed to buckling, as shown by the contours and deflection readings in Table 26.

TABLE 26.—DEFLECTIONS (INCHES) OF PLYWOOD PANELS AT FAILURE BY BUCKLING; GROUP THREE (RAISED SURFACES ARE DENOTED +)



Point[a]	Panel No. 1		Panel No. 2			Panel No. 4	Panel No. 10		Panel No. 11
Load[b]	12,960	14,940	8,950	11,920	14,940	15,920	13,000	18,000	17,700
A	....	+0 1/2	+0 3/8	....	+1 3/8	+0 1/2	....	....	0
B	....	+0 3/16	+0 1/16	....	+1 1/16	0	0 3/8	0 3/8	+0 1/4
C	....	−0 1/4	−0 1/16	....	−0 1/16	+0 1/2	....	....	+0 1/4
D	+1	+1 1/2	+0 1/4	+0 3/8	+0 3/8	−0 1/2	....	....	−0 1/4
D′	....	....	....	....	−0 3/8	....	....	....	....
E	+0 3/8	+0 1/4	+0 1/2	+0 1/2	+1 1/2	0	0 3/8	1	+0 1/4
F	+0 1/16	0	+0 1/4	+0 3/8	+0 1/2	0	....	....	0
G	+0 1/2	+0 1/2	0	....	+0 3/8	....	....	....	....
H	+1 1/2	+2	+0 1/2	....	+0 1/16	....	....	....	....
J	+0 1/16	+0 1/16	+0 1/2	....	+1 3/8	....	....	....	....
K	−0 1/2	−0 1/2	....	....	....	....	....	....	....
L	+0 1/2	+0 1/2	....	....	....	....	....	....	....
M	+0 1/2	+1 1/2	....	....	....	....	....	....	....
N	....	−0 1/2	....	....	....	....	....	....	....
P	....	−0 1/2	....	....	....	....	....	....	....
Q	....	+0 3/8	....	....	....	....	....	....	....

[a] Deflections are given for the points shown in the accompanying diagrams. [b] Loads on the panel, in pounds, at the time of observation.

Results of Tests.—The data on each test panel together with ultimate loads, manner of failures, etc., are shown in Figs. 84 and 85, and in Tables 25 and 26.

Average test values for all panels at various stud spacings are as follows:

Stud spacing (in.)	Ultimate stress (lb per lin ft)
48	1,889
24	1,875
16	2,025
Average	1,928

From these tests, it appears that a working load of 500 lb per lin ft for plywood 3/8 in. thick is allowable since it would provide a ratio R_{wa} of 3—the lowest ultimate load recorded being 1,509 lb per lin ft. Even in the panel failing at that load, failure occurred in the nails and not in the panel.

The addition of intermediate blocking seems to have had little, if any, effect upon the strength within the limited range of stud spacings and plywood thicknesses investigated.

These tests were not sufficient in number or scope to be conclusive in determining critical buckling proportions even in stud spacings as large as 4 ft.

The ultimate nail values obtained in the buckling tests are comparable with previous nail tests. For example, the average ultimate strength of a special roofing nail with $\frac{5}{16}$-in. head as found in the nail tests of group one was 505 lb per nail; and the average for the buckle tests of group three was 508 lb per nail in those tests in which the nails caused failure.

Conclusions on Plywood Tests.—The following two conclusions on the plywood tests seem justified: (*a*) the value of plywood as a diaphragm to carry large loads, even where panels are as thin as $\frac{3}{8}$ in., appears to be established beyond reasonable doubt; and (*b*) the use of stud spacings larger than the customary 16 in. or 24 in. appears to be possible.

Because of the financial limitations of this program, other contemplated features of plywood panel construction could not be investigated. These further studies are recommended to the consideration of others. Among them are the following:

I. The buckling resistance of plywood beyond the scope of the present paper;
II. The strength of plywood panels in which the edge distances of nails are $\frac{1}{2}$ in. or less, as is the case where 2-in.-wide studs are used;
III. The capacity of nails when plywood panels are nailed to each other rather than to studs or posts; and
IV. The characteristics of large plywood panels, with and without openings, using details of construction as found in current best practice, particularly where bottom sills are bolted to foundations.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

NEW FORMULAS FOR STRESSES IN CONCRETE PAVEMENTS OF AIRFIELDS

By H. M. Westergaard,[1] M. ASCE

Synopsis

The stresses investigated here are caused by loads. The load is a pressure transmitted through the oblong "footprint" of a tire of a landing gear. Three positions of this load are considered: The first is at a considerable distance from any edge or joint, in the interior of the area of a panel of the pavement; the second position is next to an edge or next to a joint that has no capacity for load transfer, but is at an appreciable distance from any corner of the panel; the third position is next to a joint that permits some transfer of load to the adjacent panel, but is at an appreciable distance from any corner of the panel. Formulas are given for stresses and deflections caused by such loads. The significant stresses are tensile stresses at the bottom of the slab. A fourth position of the load, at a corner of a panel, by which significant tensile stresses are produced at the top of the slab, is also important, but is not considered herein.

The boundary of the footprint of the tire is usually a curve that lies between an ellipse and a circumscribed rectangle, but for present purposes this curve can be replaced in most cases by an equivalent slightly more oblong ellipse, and the load can be considered to be distributed uniformly over the area of this ellipse. Specific formulas are derived under the assumption of such a load, but general formulas are also stated which apply for any shape of the footprint (including the case of dual tires).

The basic formulas for stresses under loads in the interior of the area of a panel have been published previously, but the remaining formulas are the result of new derivations.

The order of presentation is: Statement of assumptions; statement of the formulas; numerical examples; derivations; comments on evidence from tests; and conclusions.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 1, 1947.

[1] Gordon McKay Prof. of Civ. Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass.

Notation

The following notation is used:

P = load transmitted through a tire to a panel of the pavement; if a part of the total load transmitted through the tire is on the other side of a joint or outside the edge, only the part transmitted to the panel under consideration is included in P;

a, b = semiaxes of an ellipse representing the footprint of a tire; if the load is next to an edge or joint, a is the semiaxis parallel to the edge or joint; either a or b may be the greater semiaxis, depending on whether the joint is longitudinal or transverse;

x, y = horizontal rectangular coordinates; if the load is next to an edge or joint, the axis of x is along the line of the edge or joint; if the footprint of the tire is represented by an ellipse, the axis of x is in the direction of the semiaxis a;

r, θ = corresponding polar coordinates;

z = deflection;

z_e = deflection at an edge or at a joint that has no capacity for load transfer;

z_j, z'_j = deflections at the line of a joint on the two sides of the joint when the joint has some capacity for load transfer;

σ_x, σ_y = tensile stresses at the bottom of the slab in the directions of x and y in the interior of the area of the panel;

σ_e = tensile stress at the bottom of the slab along an edge or along a joint that has no capacity for load transfer;

σ_j, σ'_j = tensile stresses along a joint at the bottom of the slabs on the two sides of the joint when the joint has some capacity for load transfer;

h = thickness of the slab, assumed constant;

E = modulus of elasticity of the concrete in the slab, assumed constant;

μ = Poisson's ratio of the concrete;

k = modulus of subgrade reaction, assumed to be a constant, measurable in pounds per cubic inch, and defining the reaction under the slab per unit of area as $k\,z$;

l = radius of relative stiffness, defined as:

$$l^4 = \frac{E\,h^3}{12\,(1 - \mu^2)\,k} \qquad (1)$$

Natural logarithms are used unless otherwise stated.

Assumptions

The assumptions usually made in the study of slabs on a subgrade are: (1) Within the range of action the slab is elastic with single constant values of the modulus of elasticity E and Poisson's ratio μ; (2) the thickness, h, of the slab is constant; and (3) the reaction of the base on the slab is a vertical pressure, equal, per unit of area, to a constant k times the deflection z, the base being uniform in character and everywhere in contact with the slab. The con-

stant k is called the modulus of subgrade reaction and is measurable in pounds per cubic inch. It is kept in mind that the third assumption is an empirical makeshift, which however has been found in the past to give usable results.

It is assumed furthermore—and this can be assumed without hesitation—that the average of the width and length of the footprint of the tire is greater than the thickness of the slab in all significant cases. Under this assumption, the ordinary theory of bending of plates or slabs can be used without the special corrections that would be needed if the loaded area were very small, as it sometimes must be considered to be in the corresponding problems of concrete highways.

Each panel of pavement is assumed to be fairly large and as yet uncracked. The load is assumed in no case to be close to more than one edge or joint; other edges or joints are assumed to be far enough away to have only a negligible influence on the stresses directly under the load.

Statement of the Formulas

A. Stresses and Deflections in the Interior of the Area of a Panel.—

Case 1.—The load is applied in the interior of the area of the panel at a considerable distance from any edge or joint. The total load P is distributed uniformly over the area of the ellipse—

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots\ldots(2)$$

The principal tensile stresses σ_x and σ_y, in the directions of x and y, at the bottom of the slab under the center of the load may be computed either by the formula—

$$\left.\begin{matrix}\sigma_x \\ \sigma_y\end{matrix}\right\} = \frac{3\,P}{8\,\pi\,h^2}\left[(1+\mu)\log\frac{E\,h^3}{k\left(\frac{a+b}{2}\right)^4} \mp 2\,(1-\mu)\,\frac{a-b}{a+b}\right] \quad \ldots\ldots(3)$$

or by the equivalent formula—

$$\left.\begin{matrix}\sigma_x \\ \sigma_y\end{matrix}\right\} = \frac{P}{h^2}\left[0.275\,(1+\mu)\log_{10}\frac{E\,h^3}{k\left(\frac{a+b}{2}\right)^4} \mp 0.239\,(1-\mu)\,\frac{a-b}{a+b}\right] \quad \ldots(4)$$

The deflection at any point x,y within or near the loaded area may be computed either by the formula—

$$z = \frac{P}{8\,k\,l^2}\left[1 - \frac{a^2+b^2+4\,x^2+4\,y^2}{16\,\pi\,l^2}\log\frac{E\,h^3}{k\left(\frac{a+b}{2}\right)^4} - \frac{a^2+4\,a\,b+b^2}{16\,\pi\,l^2} + \frac{(a-b)\,(x^2-y^2)}{2\,\pi\,l^2\,(a+b)}\right] \quad \ldots(5)$$

(in which l is defined by Eq. 1) or by the equivalent formula—

$$z = \frac{P}{4}\sqrt{\frac{3\,(1-\mu^2)}{E\,h^3\,k}} - \frac{0.275\,(1-\mu^2)\,P}{E\,h^3}\left[\frac{a^2+b^2}{4}+x^2+y^2\right]\log_{10}\frac{E\,h^3}{k\left(\frac{a+b}{2}\right)^4} - \frac{0.239\,(1-\mu^2)\,P}{E\,h^3}\left[\frac{a^2+4\,a\,b+b^2}{8}+\frac{a-b}{a+b}\,(-x^2+y^2)\right] \quad \ldots\ldots(6)$$

Case 2.—The load again is applied in the interior of the area of the panel at a considerable distance from any edge or joint, but the load now has a more general type of distribution. The total load P is distributed uniformly over any area A having both axes of x and y as axes of symmetry. Such an area may be the footprint of either a single or a dual tire.

The principal stresses σ_x and σ_y at the bottom of the slab under the center of the load may be computed by the following formula:

$$\left.\begin{matrix}\sigma_x\\ \sigma_y\end{matrix}\right\} = \frac{3\,P}{2\,\pi\,h^2}\left[(1+\mu)\,(K+0.1159) \pm \frac{1-\mu}{2}\,S\right] \quad \ldots\ldots(7)$$

in which K and S are "area coefficients" defined by the equations:[2]

$$K = -\frac{1}{A}\int dA\,\log\frac{r}{l} \quad \ldots\ldots(8)$$

and

$$S = -\frac{1}{A}\int dA\,\cos 2\,\theta \quad \ldots\ldots(9)$$

The integrations in Eqs. 8 and 9 are extended over the entire loaded area, and l is defined by Eq. 1.

The deflection at any point x,y within or near the loaded area may be computed by the formula:

$$z = \frac{P}{8\,k\,l^2} - \frac{P}{8\,\pi\,k\,l^4}\left[\frac{1}{A}\int r^2\,dA\,\log\frac{l}{r} + \frac{1.1159}{A}\int r^2\,dA + (K+0.1159)\,(x^2+y^2) + \tfrac{1}{2}\,S\,(x^2-y^2)\right] \quad \ldots\ldots(10)$$

B. Stresses and Deflections at an Edge or at a Joint That Has No Capacity for Load Transfer.—

Case 3.—The load is next to an edge or next to a joint that has no capacity for load transfer. The axis of x is along the edge or joint. The total load P is distributed uniformly over the area of an ellipse which is tangent to the edge or joint and has the equation:

$$\frac{x^2}{a^2}+\frac{(y-b)^2}{b^2} = 1 \quad \ldots\ldots(11)$$

[2] "Stress Concentrations in Plates Loaded Over Small Areas," by H. M. Westergaard, *Transactions*, ASCE, Vol. 108, 1943, pp. 831–856, especially p. 835, Eqs. 21 and 22.

The tensile stress at the bottom along the edge or joint directly under the point of tangency of the ellipse, that is, at $x = y = 0$, may be computed either by the formula—

$$\sigma_e = \frac{3\,(1+\mu)\,P}{\pi\,(3+\mu)\,h^2}\left[\log\frac{E\,h^3}{100\,k\left(\frac{a+b}{2}\right)^4} + 1.84 - \frac{4}{3}\,\mu + (1+\mu)\,\frac{a-b}{a+b} + 2\,(1-\mu)\,\frac{a\,b}{(a+b)^2} + 1.18\,(1+2\,\mu)\,\frac{b}{l}\right] \ldots\ldots(12)$$

or by the equivalent formula—

$$\sigma_e = \frac{2.2\,(1+\mu)\,P}{(3+\mu)\,h^2}\log_{10}\frac{E\,h^3}{100\,k\left(\frac{a+b}{2}\right)^4} + \frac{3\,(1+\mu)\,P}{\pi\,(3+\mu)\,h^2}\left[1.84 - \frac{4}{3}\,\mu + (1+\mu)\,\frac{a-b}{a+b} + 2\,(1-\mu)\,\frac{a\,b}{(a+b)^2} + 1.18\,(1+2\,\mu)\,\frac{b}{l}\right] \ldots(13)$$

in which l, as elsewhere, is the radius of relative stiffness defined by Eq. 1.

The deflection at any point on the axis of symmetry perpendicular to the edge or joint at a moderate distance y from the edge or joint (or at the edge or joint itself) may be computed by the approximate formula:

$$z = \frac{P\sqrt{2+1.2\,\mu}}{\sqrt{E\,h^3\,k}}\left[1 - (0.76 + 0.4\,\mu)\,\frac{b}{l}\right]\left[1 - (0.76 + 0.4\,\mu)\,\frac{y}{l}\right] \ldots(14)$$

The character of Eq. 14 as an approximate formula is indicated by the omission of quadratic terms in y, or in a and b. These quadratic terms are relatively unimportant when both b and y have moderate values.

Case 4.—As in Case 3, the load is again next to an edge or next to a joint that has no capacity for load transfer, but the total load P is now distributed uniformly over the area of one half of an ellipse. The ellipse has the principal radii a and b, and the diameter $2\,a$ is along the edge or joint. This condition may occur when one half of the footprint of the tire is outside the panel under consideration; P is then one half of the total pressure transmitted through the tire.

The tensile stress at the bottom of the slab along the edge or joint under the center of the ellipse may be computed by the following formula:

$$\sigma_e = \frac{2.2\,(1+\mu)\,P}{(3+\mu)\,h^2}\log_{10}\frac{E\,h^3}{100\,k\left(\frac{a+b}{2}\right)^4} + \frac{3\,(1+\mu)\,P}{\pi\,(3+\mu)\,h^2}\left[3.84 - \frac{4}{3}\,\mu - (1-\mu)\,\frac{a-b}{a+b} + 0.5\,(1+2\,\mu)\,\frac{b}{l}\right] \ldots(15)$$

The deflection at the center of the ellipse or at any other point on the axis of symmetry at a moderate distance y from the edge or joint may be computed by the approximate formula:

$$z = \frac{P\sqrt{2 + 1.2\,\mu}}{\sqrt{E\,h^3\,k}}\left[1 - (0.323 + 0.17\,\mu)\frac{b}{l}\right]\left[1 - (0.76 + 0.4\,\mu)\frac{y}{l}\right] \ldots (16)$$

Case 5.—As in Cases 3 and 4, the load is next to an edge or next to a joint that has no capacity for load transfer, but, as in Case 2, the load has a more general type of distribution. The total load P is distributed uniformly over any area A having the axis of y as an axis of symmetry. The axis of x is along the edge or joint.

The tensile stress at the bottom of the slab along the edge or joint at the point $x = y = 0$ may be computed by the formula:

$$\sigma_e = \frac{3\,(1 + \mu)\,P}{\pi\,(3 + \mu)\,h^2}\left[4\,K - 0.28 - \frac{4}{3}\mu - \mu^2 + (1 - \mu)\,S + 1.18\,(1 + 2\,\mu)\frac{\bar{y}}{l}\right] \ldots (17)$$

in which $\bar{y}$ is the distance from the edge or joint to the center of gravity of the load, l is the radius of relative stiffness defined by Eq. 1, and K and S are the area coefficients defined by Eqs. 8 and 9. In computing K and S by these formulas it should be noted that in this case, unlike Case 2, the axis of x, being along the edge or joint, is not an axis of symmetry.

The deflection at any point on the axis of symmetry perpendicular to the edge or joint at a moderate distance y from the edge or joint (or at the edge or joint itself) may be computed by the approximate formula:

$$z = \frac{P\sqrt{2 + 1.2\,\mu}}{\sqrt{E\,h^3\,k}}\left[1 - (0.76 + 0.4\,\mu)\frac{\bar{y}}{l}\right]\left[1 - (0.76 + 0.4\,\mu)\frac{y}{l}\right] \ldots (18)$$

C. Stresses and Deflections at a Joint That Has Some Capacity for Load Transfer.—

Case 6.—The load is at a joint and may be partly on one side and partly on the other side of the joint. The transfer devices are either continuous along the joint or are closely spaced. The joint is capable of transferring load with an efficiency j which may have any value in the range $0 < j \leqq 1$. The extreme value $j = 1$ represents 100% efficiency, which means that the deflections of the two adjacent slabs are certain to be equal at the line of the joint. The other extreme $j = 0$ would mean no transfer of load as in the preceding Cases 3 to 5. The meaning given here to any intermediate value of j is expressed by a formula which is recognized to be approximate only, but which nevertheless will serve the present purpose of estimating the relief of stress by load transfer; the formula is:

$$z_j - z'_j = (1 - j)\,(z_e - z'_e) \ldots (19)$$

in which z_j and z'_j are the deflections of the adjacent slabs at any point at the joint, while z_e and z'_e are the corresponding deflections that would occur at the

same place if the joint had no capacity for load transfer. Eq. 19 must be considered as approximate only, because j is assumed to be constant over some distance.

Under these assumptions the tensile stresses σ_i and σ'_i at the bottom of the two slabs along the joint may be computed by the formulas:

$$\sigma_i = (1 - \tfrac{1}{2} j)\, \sigma_e + \tfrac{1}{2} j\, \sigma'_e \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

and

$$\sigma'_i = \tfrac{1}{2} j\, \sigma_e + (1 - \tfrac{1}{2} j)\, \sigma'_e \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which σ_e and σ'_e are the corresponding stresses that would be produced if the joint had no capacity for load transfer. That is, depending on the distribution of the load, σ_e and σ'_e may be defined by such formulas as Eqs. 13, 15, or 17.

Similarly, the deflections of the two slabs at the joint may be computed by the formulas:

$$z_i = (1 - \tfrac{1}{2} j)\, z_e + \tfrac{1}{2} j\, z'_e \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

and

$$z'_i = \tfrac{1}{2} j\, z_e + (1 - \tfrac{1}{2} j)\, z'_e \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

It is observed that Eqs. 22 and 23 obey Eq. 19.

Numerical Examples

The following values are selected as common to all the numerical examples:

(*a*) The total load transmitted through one tire is 50,000 lb. The footprint of the tire is taken as an ellipse with an average diameter $a + b = 30$ in. Either a or b is 20 in.

(*b*) The thickness of the pavement is $h = 10$ in.

(*c*) The modulus of elasticity and Poisson's ratio of the concrete are $E = 4{,}000{,}000$ lb-in.$^{-2}$ and $\mu = 0.15$. The modulus of subgrade reaction is $k = 200$ lb-in.$^{-3}$

With these values one finds the following constants which recur in the computations; the first being the radius of relative stiffness defined by Eq. 1: $l = 36.14$ in.; $l^2 = 1{,}305.8$ in.^{2}; and

$$\log_{10} \frac{E\, h^3}{100\, k \left(\dfrac{a+b}{2}\right)^4} = 0.5967 \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Example 1.—The load is in the interior of the area of the panel. The semi-axes of the loaded ellipse are $a = 20$ in. and $b = 10$ in.

The stresses σ_x and σ_y under the center of the load in the directions of the longer and shorter axis, respectively, are computed by Eq. 4 as

$$\left.\begin{matrix}\sigma_x \\ \sigma_y\end{matrix}\right\} = (410.6 \mp 33.9) \text{ lb-in.}^{-2} = \left\{\begin{matrix}376.7 \text{ lb-in.}^{-2} \\ 444.5 \text{ lb-in.}^{-2}\end{matrix}\right. \ldots\ldots\ldots\ldots (25)$$

The three terms in Eq. 6 and the resultant deflection under the center of

the load at $x = y = 0$ are stated as follows:

$$z = (0.02393 - 0.00109 - 0.00047) \text{ in.} = 0.02237 \text{ in.} \quad (26)$$

The same results are obtained from Eq. 5.

At the end of the major axes, at $x = a, y = 0$, the value in Eq. 26 is changed, according to Eq. 6, by multiplying the second term by $\frac{21}{5}$ and the third by $\frac{7}{39}$. This gives the deflection at that point as $z = 0.01927$ in.

Example 2.—The load is next to a longitudinal joint that has no capacity for load transfer. The loaded elliptic area is tangent to the joint. The semiaxes are $a = 20$ in. and $b = 10$ in.

The stress at the joint is computed by Eq. 13. The two terms and the resulting value are as follows:

$$\sigma_e = (239.6 + 501.2) \text{ lb-in.}^{-2} = 740.8 \text{ lb-in.}^{-2} \quad (27)$$

Eq. 14 gives the following values of the deflections z_e at the joint and z under the center of the load:

$$z_e = 0.0638 \text{ in.} \quad \text{and} \quad z = 0.0493 \text{ in.} \quad (28)$$

Example 3.—The load is next to a transverse joint that has no capacity for load transfer. The loaded elliptic area is tangent to the joint. The semiaxes are $a = 10$ in. and $b = 20$ in. The values in Eqs. 27 and 28 are changed by the exchange of the values of a and b to

$$\sigma_e = 663.8 \text{ lb-in.}^{-2}, \quad z_e = 0.045 \text{ in.}, \quad \text{and} \quad z = 0.025 \text{ in.} \quad (29)$$

but the last of these values should be considered only as a crude approximation because the distance of 20 in. from the joint to the center of the load is rather large for this computation.

Example 4.—The load stradles a longitudinal joint, with half on each side; $a = 20$ in. and $b = 10$ in. The load on one side of the joint is 25,000 lb. The efficiency of the joint does not enter into the computation. Eqs. 15 and 16 give the stress and the deflection at the joint as

$$\sigma_e = 428.0 \text{ lb-in.}^{-2} \quad \text{and} \quad z_e = 0.0373 \text{ in.} \quad (30)$$

It is noted that the stress is a little less than σ_y in Example 1, in which the load is in the interior of the area, but the deflection is conspicuously greater.

Example 5.—The load is next to a longitudinal joint which has an efficiency $j = 0.8$ (or 80%) for load transfer. The loaded elliptic area is tangent to the joint. The semiaxes are $a = 20$ in. and $b = 10$ in. With σ_e taken from Eq. 27, Eqs. 20 and 21 give the following stresses on the two sides of the joint:

$$\sigma_j = 740.8 \text{ lb-in.}^{-2} (1 - \tfrac{1}{2} 0.8) = 444.5 \text{ lb-in.}^{-2} \quad \text{and} \quad \sigma'_j = 296.3 \text{ lb-in.}^{-2} \quad (31)$$

Similarly Eqs. 22, 23, and 28 give the deflections on the two sides of the joint:

$$z_j = 0.0383 \text{ in.} \quad \text{and} \quad z'_j = 0.0255 \text{ in.} \quad (32)$$

It is observed that σ_j in Eqs. 31 has the same value as σ_y in Eq. 25. This indicates that, under the circumstances, a joint efficiency of 80% or more would be desirable.

Example 6.—The load is next to a transverse joint which has an efficiency $j = 0.66$ for load transfer. The loaded elliptic area is tangent to the joint. The semiaxes are $a = 10$ in. and $b = 20$ in. Eqs. 20, 22, and 29 give:

$$\sigma_j = 444.7 \text{ lb-in.}^{-2} \quad \text{and} \quad z_j = 0.030 \text{ in.} \ldots\ldots\ldots\ldots (33)$$

Derivations

A. Stresses and Deflections in the Interior of the Area of a Panel.—Eqs. 7, 8, and 9, which define the stresses when the loaded area has any shape, are taken from a former[3] paper by the writer, in which the use of area coefficients such as K and S in Eqs. 8 and 9, and place coefficients such as the constant 0.1159 in Eq. 7 were introduced.[3] Eq. 3 defines the stresses in the interior of the area when the loaded area is the ellipse in Eq. 2. It is obtained from Eq. 7 by substituting l from Eq. 1, omitting within the brackets a small positive term containing μ^2 and a nearly balancing negative term. By substituting the following values[4] for K and S:

$$K = \log \frac{2\,l}{a+b} + \tfrac{1}{2} \quad \text{and} \quad S = -\frac{a-b}{a+b} \ldots\ldots\ldots (34)$$

Eq. 4 is a transcription of Eq. 3. With $a = b$ (that is, when the ellipse is a circle), Eq. 4 agrees with formulas published earlier.[5,6]

In a paper published in 1940,[7] the writer showed that even with a fairly large circular footprint of the tire the correction that would have to be added to the stress in the original formulas because of the large radius would be small. For example, with $a = b = 15$ in., but with other values as in the present numerical Example 1, the added stress would be only 4.6 lb-in.$^{-2}$. Such a small correction may well be ignored, and the correction of this type has been ignored in Eqs. 3, 4, and 7.

The 1940 paper[8] also contains a formula for the deflection at the center of a loaded circular area. That formula agrees with, and thus confirms, Eqs. 5 and 6 when these equations are applied with $a = b$ and $x = y = 0$; but Eqs. 5 and 6 apply when the loaded area is an ellipse. Eq. 5 is derived from Eq. 10 which applies when the loaded area is of a general type, but symmetrical about both axes of x and y. In performing this derivation the first integral in Eq. 10 can be obtained in a finite form; the second integral is merely the polar moment of inertia of the area; and K and S are given in Eqs. 34.

It remains to show the validity of Eq. 10. The terms containing x^2 and y^2 are verified by noting that they define the same curvatures as do the stresses in Eq. 7. On account of the symmetry of the load there can be no linear terms in x and y. The remaining terms in Eq. 10 are correct if the deflection due to

[3] "Stress Concentrations in Plates Loaded Over Small Areas," by H. M. Westergaard, *Transactions*, ASCE, Vol. 108, 1943, p. 834 et seq., especially Eqs. 15, 21, 22, and 51.

[4] *Ibid.*, pp. 840 and 841, Eqs. 51, 65, and 66.

[5] "Stresses in Concrete Pavements Computed by Theoretical Analysis," by H. M. Westergaard, *Public Roads*, April, 1926, pp. 25–35, especially Eq. 10.

[6] "Analytical Tools for Judging Results of Structural Tests of Concrete Pavements," by H. M. Westergaard, *ibid.*, December, 1933, pp. 185–188, especially Eq. 4.

[7] "Stresses in Concrete Runways of Airports," by H. M. Westergaard, *Proceedings*, 19th Annual Meeting, Highway Research Board, December, 1939 (published in 1940), pp. 197–202, especially Eqs. 6, 7, and 50.

[8] *Ibid.*, p. 199, Eq. 11.

a concentrated load P at the distance r from the point of application of P can be stated by the formula:

$$z = \frac{P}{8\,k\,l^2} - \frac{P\,r^2}{8\,\pi\,k\,l^4}\left(\log\frac{l}{r} + 1.1159\right) \quad \ldots\ldots (35)$$

Eq. 35 is obtained from the exact solution,[9,10] by omitting terms containing r^4 and higher powers of r. This concludes the verification of Eqs. 5, 6, and 10.

B. Stresses and Deflections at an Edge or at a Joint That Has No Capacity for Load Transfer.—

The basis of Eqs. 17 and 18, which apply when the load is next to an edge and is distributed over a not too large area of any shape, is two formulas which are taken from a paper published in 1923.[11] The first defines the deflection z produced at the point x,y by a load P at $x = y = 0$ when the axis of x is along an edge and the slab covers the entire region $y > 0$. The second formula, which is used as an auxiliary equation, defines in a related form the deflection z_i, produced at the point x,y with $y \geqq 0$ by a load P at $x = y = 0$ when the slab covers the entire xy-plane. The formulas are:

$$z = \frac{2\,P}{\pi\,k\,l^2}\int_0^\infty \frac{\gamma\cos\frac{\alpha\,x}{l}\left[\cos\frac{\beta\,y}{l} + (1-\mu)\,\alpha^2\sin\frac{\beta\,y}{l}\right]e^{-\gamma y/l}\,d\alpha}{1 + 4\,(1-\mu)\,\alpha^2\,\gamma^2 - (1-\mu)^2\,\alpha^4} \quad \ldots (36)$$

and

$$z_i = \frac{P}{2\,\pi\,k\,l^2}\int_0^\infty \frac{\cos\frac{\alpha\,x}{l}\left[\beta\cos\frac{\beta\,y}{l} + \gamma\sin\frac{\beta\,y}{l}\right]e^{-\gamma y/l}\,d\alpha}{\beta^2 + \gamma^2} \quad \ldots\ldots (37)$$

in which α, β, and γ are three positive values obeying the relations:

$$\alpha^2 + \beta^2 = \gamma^2 \qquad \text{and} \qquad 2\,\beta\,\gamma = 1 \quad \ldots\ldots (38)$$

Eqs. 36 and 37 can be verified by showing that they both satisfy the governing differential equation—

$$l^4\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 z + z = 0 \quad \ldots\ldots (39)$$

both give values converging toward zero as the distance goes to infinity; and both account for the load and satisfy the appropriate conditions at $y = 0$.

According to the Maxwell theorem of reciprocal deflections, z in Eq. 36 may also be interpreted as the deflection produced at the origin of the coordinates by a load P at point x,y. It follows that the diagram of deflections produced at the edge, at $y = 0$, by a load P at point $0,y$ may be obtained as the diagram of deflections produced at the distance y from the edge by a load P at the origin. It follows from this that the curvature $-\frac{\partial^2 z}{\partial x^2}$ produced at the origin by a load

[9] "Stress Concentrations in Plates Loaded Over Small Areas," by H. M. Westergaard, *Transactions*, ASCE, Vol. 108, 1943, p. 839, Eq. 48.

[10] "Stresses in Concrete Runways of Airports," by H. M. Westergaard, *Proceedings*, 19th Annual Meeting, Highway Research Board, December, 1939 (published in 1940), p. 197, Eqs. 30 and 34.

[11] "Om Beregning af Plader paa elastisk Underlag med særligt Henblik paa Spørgsmaalet om Spændinger i Betonveje," by H. M. Westergaard, *Ingeniøren*, Vol. 32, 1923, pp. 513–524, especially Eqs. 47, 30, and 34, and also Eqs. 48 and 33.

P at point x,y may be obtained as the curvature $-\frac{\partial^2 z}{\partial x^2}$ produced at point x,y by a load P at the origin of the coordinates.

The curvature in the direction of x at the origin of the coordinates defines the tensile stress along the bottom of the edge at that point as:

$$\sigma_e = -\frac{E\,h}{2}\frac{\partial^2 z}{\partial x^2} \quad \ldots\ldots (40)$$

According to the dual interpretation of the curvatures, if the load P is at point x,y, the function z in Eq. 40 is also the function z defined by Eq. 36. By use of Eq. 1 this gives the stress at the edge due to P at point x,y as:

$$\sigma_e = \frac{12\,(1-\mu^2)\,P}{\pi\,h^2}\int_0^\infty \frac{\alpha^2\,\gamma\cos\frac{\alpha\,x}{l}\left[\cos\frac{\beta\,y}{l}+(1-\mu)\,\alpha^2\sin\frac{\beta\,y}{l}\right]e^{-\gamma y/l}\,d\alpha}{1+4\,(1-\mu)\,\alpha^2\gamma^2-(1-\mu)^2\,\alpha^4} \quad \ldots\ldots (41)$$

The function σ_e in Eq. 41 has a singularity at $x = y = 0$, with an infinite value at that point. The difficulty involved in this is overcome by dividing σ_e into two component parts, writing

$$\sigma_e = \bar{\sigma} + \sigma_c \quad \ldots\ldots (42)$$

and choosing the part $\bar{\sigma}$ so that it will have the same type of singularity as σ_e but will be of a simpler form. This leaves σ_c as a smooth function with a finite value at the origin of the coordinates.

The choice of the function $\bar{\sigma}$ is aided by noting the type of the singularity that the elastic surface of any plate must have at the point of application of a concentrated force applied at a free edge. A. Nádai[12] has defined that type. In this way one arrives at the following possible form of $\bar{\sigma}$:

$$\bar{\sigma} = -\frac{12\,(1+\mu)\,k\,l^4}{(3+\mu)\,h^2}\left[(3-\mu)\frac{\partial^2 z_i}{\partial x^2}+(1+\mu)\frac{\partial^2 z_i}{\partial y^2}\right] \quad \ldots\ldots (43)$$

in which z_i is the deflection produced at point x,y by a load P at the origin of the coordinates in a slab that covers the entire x,y-plane. The function z_i can be defined by Eq. 37; but in the vicinity of the origin of the coordinates the two second derivatives in Eq. 43 may also be expressed in the manner of Eq. 7 by the formula:[13]

$$\left.\begin{matrix} -\frac{\partial^2 z}{\partial x^2} \\ -\frac{\partial^2 z}{\partial y^2}\end{matrix}\right\} = \frac{P}{4\,\pi\,k\,l^4}\left(\log\frac{l}{r}+0.1159 \mp \tfrac{1}{2}\cos 2\,\theta\right) \quad \ldots\ldots (44)$$

This gives

$$\bar{\sigma} = \frac{12\,(1+\mu)\,P}{\pi\,(3+\mu)\,h^2}\left(\log\frac{l}{r}+0.1159-\frac{1-\mu}{4}\cos 2\,\theta\right) \quad \ldots\ldots (45)$$

The remaining part of σ_e is obtained as

$$\sigma_c = \sigma_e - \bar{\sigma} \quad \ldots\ldots (46)$$

[12] "Die elastischen Platten," by A. Nádai, Julius Springer, Berlin, 1925, p. 204.

[13] "Stress Concentrations in Plates Loaded Over Small Areas," by H. M. Westergaard, *Transactions*, ASCE, Vol. 108, 1943, p. 835 et seq., Eqs. 17, 21, 22, and 51.

with σ_e as in Eq. 41 and with $\bar{\sigma}$ again defined by Eq. 43, but this time with z_i substituted from Eq. 37, which thereby serves its purpose as an auxiliary formula. This gives:

$$\sigma_e = -\frac{24\,(1+\mu)\,P}{\pi\,(3+\mu)\,h^2}\int_0^\infty \frac{\cos\frac{\alpha\,x}{l}\,[1+(1-\mu)^2\,\alpha^2\,\beta^2]\,\beta\cos\frac{\beta\,y}{l}\,e^{-\gamma y/l}\,d\alpha}{1+4\,(1-\mu)\,\alpha^2\,\gamma^2-(1-\mu)^2\,\alpha^4}$$

$$-\frac{12\,(1-\mu^2)\,P}{\pi\,(3+\mu)\,h^2}\int_0^\infty \frac{\cos\frac{\alpha\,x}{l}\,[1+4\,(1-\mu)\,\alpha^2\,\beta^2]\,\gamma\sin\frac{\beta\,y}{l}\,e^{-\gamma y/l}\,d\alpha}{1+4\,(1-\mu)\,\alpha^2\,\gamma^2-(1-\mu)^2\,\alpha^4}$$

$$+\frac{6\,(1+\mu)\,(3-\mu)\,P}{\pi\,(3+\mu)\,h^2}\int_0^\infty \frac{\cos\frac{\alpha\,x}{l}\left[2\,\beta^3\cos\frac{\beta\,y}{l}+\beta\sin\frac{\beta\,y}{l}\right]e^{-\gamma y/l}\,d\alpha}{\beta^2+\gamma^2} \quad \ldots(47)$$

in which, as before, α, β, and γ are defined by Eqs. 38.

As was anticipated, σ_e in Eq. 47 is in fact a smooth function without infinite values in the half plane $y \geqq 0$. In the region of small values of x and y the function σ_e may be written with adequate accuracy as a linear function of y in the form:

$$\sigma_e = [\sigma_e]_{x=y=0} + y\left[\frac{\partial\sigma_e}{\partial y}\right]_{x=y=0} \quad \ldots(48)$$

In evaluating the constant term in Eq. 48 it is noted that:

$$\int_0^\infty \frac{\beta^3\,d\alpha}{\beta^2+\gamma^2} = \tfrac{1}{4} \quad \ldots(49)$$

This leads to the following form of Eq. 48:

$$\sigma_e = \frac{12\,(1+\mu)\,P}{\pi\,(3+\mu)\,h^2}\left(\frac{3-\mu}{4} - B_1 + B_2\,\frac{y}{l}\right) \quad \ldots(50)$$

in which B_1 and B_2 are dimensionless constants defined by the formulas:

$$B_1 = 2\int_0^\infty \frac{[1+(1-\mu)^2\,\alpha^2\,\beta^2]\,\beta\,d\alpha}{1+4\,(1-\mu)\,\alpha^2\,\gamma^2-(1-\mu)^2\,\alpha^4} \quad \ldots(51)$$

and

$$B_2 = \tfrac{1}{2}\int_0^\infty \frac{[1+\mu-2\,(1-\mu)^2\,\alpha^2\,\beta^2]\,d\alpha}{1+4\,(1-\mu)\,\alpha^2\,\gamma^2-(1-\mu)^2\,\alpha^4} \quad \ldots(52)$$

When the load P is distributed over an area A instead of being concentrated at the point x,y, Eq. 45 is replaced by

$$\bar{\sigma} = \frac{12\,(1+\mu)\,P}{\pi\,(3+\mu)\,h^2}\left(K + 0.1159 + \frac{1-\mu}{4}\,S\right) \quad \ldots(53)$$

in which K and S are the place coefficients defined by Eqs. 8 and 9; and Eq. 50 is modified by replacing y by the distance $\bar{y}$ from the edge to the center of gravity of the load. Then Eqs. 42, 50, and 53 lead to the following formula

for the tensile stress along the bottom of the edge at the point $x = y = 0$ due to the actual load:

$$\sigma_e = \frac{12\,(1+\mu)\,P}{\pi\,(3+\mu)\,h^2}\left[\,K + 0.8659 - \frac{\mu}{4} - B_1 + \frac{1-\mu}{4}\,S + B_2\frac{\bar{y}}{l}\,\right] \ldots (54)$$

in which B_1 and B_2 are defined by Eqs. 51 and 52.

Like the part σ_e of the stress, the deflection z_e at the edge at $x = y = 0$ can be expressed with adequate accuracy as a linear function of the distance $\bar{y}$ from the edge, provided that the loaded area is not too great. Thus one may write

$$z_e = [z]_{x=y=0} + \bar{y}\left[\frac{\partial z}{\partial y}\right]_{x=y=0} \ldots\ldots\ldots\ldots\ldots\ldots (55)$$

or, in terms of two dimensionless constants B_3 and B_4,

$$z_e = \frac{P}{k\,l^2}\left[\,B_3 - B_4\frac{\bar{y}}{l}\,\right] \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

The values of B_3 and B_4 are determined through Eqs. 36 and 55 and are found to be

$$B_3 = \frac{1}{M\sqrt{8}}\left(\frac{M+1-\mu}{\sqrt{2-\mu+M}} + \frac{M-1+\mu}{\sqrt{2-\mu-M}}\right) \ldots\ldots\ldots (57)$$

in which the quantity M is defined as

$$M = \sqrt{(1-\mu)^2 + \mu^2} \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

and

$$B_4 = \frac{1}{\pi}\int_0^\infty \frac{[(1+\mu)\,\alpha^2 + 2\,\beta^2]\,d\alpha}{1 + 4\,(1-\mu)\,\alpha^2\,\gamma^2 - (1-\mu)^2\,\alpha^4} \ldots\ldots\ldots (59)$$

in which, as before, α, β, and γ are positive values defined by Eqs. 38.

Numerical values of the constants B_1, B_2, B_3, and B_4, as defined by Eqs. 51, 52, 57, and 59, and computed by members of the staff of the David Taylor Model Basin, United States Navy Department, at Carderock, Md., are given in Table 1. Numerical studies of these values have led to the following formulas through which the four constants are determined with quite satisfactory accuracy within the range of μ, and are given in forms that lend themselves to the final formulas:

TABLE 1.—VALUES OF THE CONSTANTS B_1, B_2, B_3, AND B_4 IN EQS. 54 AND 56 FOR DIFFERENT VALUES OF POISSON'S RATIO μ

μ	B_1	B_2	B_3	B_4
0 ...	0.9352	0.2952	0.4082	0.3118
0.15 ...	0.9544	0.3822	0.4314	0.3510
0.2 ...	0.9627	0.4131	0.4406	0.3715
0.3 ...	0.9830	0.4786	0.4620	0.4106

$$4\,(B_1 - 0.8659) = 0.28 + \tfrac{1}{3}\,\mu + \mu^2 \ldots\ldots\ldots\ldots (60)$$

$$4\,B_2 = 1.18\,(1 + 2\,\mu) \ldots\ldots\ldots\ldots\ldots\ldots (61)$$

$$B_3\sqrt{12\,(1-\mu^2)} = \sqrt{2 + 1.2\,\mu} \ldots\ldots\ldots\ldots\ldots (62)$$

$$B_4 = (0.76 + 0.4\,\mu)\,B_3 \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

Substitution from Eqs. 60 and 61 in Eq. 54 leads directly to Eq. 17 which defines σ_e for any shape of the loaded area. Eqs. 12, 13, and 15 are specific forms of Eq. 17, which are derived by use of Eq. 1, by replacing $\log (1 - \mu^2)$ by $-\mu^2$, and by substituting appropriate values of K and S; namely, for Eq. 15 the values in Eqs. 34, and for Eqs. 12 and 13 the following values which apply at the ends of the axis $2b$ of the elliptic area:[4]

$$K = \log \frac{2l}{a+b} + \frac{a-b}{2(a+b)} \qquad \text{and} \qquad S = -\frac{a-b}{a+b} + \frac{2ab}{(a+b)^2} \ldots (64)$$

It should be mentioned that Eq. 15, which applies when the load is distributed over one half of an ellipse, does not become identical, when $a = b$, to the formula—given elsewhere[4]—for the case of a load distributed over a semicircle.[14,15] The stresses defined by Eq. 15 are greater than those defined by the previous formula.[14,15]

The value of z defined by Eq. 18 when $y = 0$ is derived from Eq. 56 by substitution from Eqs. 62 and 63 and by use of Eq. 1. The second quantity in brackets in Eq. 18, which extends the applicability of the formula to points near the edge, is written so as to make the complete formula conform to the Maxwell theorem of reciprocal deflections. Eqs. 14 and 16 are merely specific forms of Eq. 18.

This completes the verification of the formulas for stresses and deflections at an edge.

C. Stresses and Deflections at a Joint That Has Some Capacity for Load Transfer.—

Assume that the load P is entirely on side A of the joint. For the purpose of the analysis this load can be resolved into two component parts: Component I consists of $\frac{1}{2}P$ on side A and $\frac{1}{2}P$ placed symmetrically on side B; component II consists of $\frac{1}{2}P$ on side A and $-\frac{1}{2}P$ on side B. Component I produces the deflections $\frac{1}{2}z_e$ on each side. Component II produces equal and opposite deflections on the two sides. According to Eq. 19 these deflections are $\pm\frac{1}{2}(1 - j)z_e$. This leads to the resultant deflections defined by Eqs. 22 and 23 with $z'_e = 0$. The complete Eqs. 22 and 23 are obtained by superposition of the effects of loads on both sides. Eqs. 20 and 21 which define the stresses are consistent with Eqs. 22 and 23.

Comments on the Evidence Obtained from Tests and Experience

Since the 1920's experimental evidence has accumulated supporting the claim of usability of the formulas published previously for stresses in concrete highways under loads distributed over a small circular area. The Public Roads Administration—formerly The Bureau of Public Roads—has conducted many of the tests. A comprehensive report in five parts published in 1935, 1936, and 1943 by L. W. Teller and Earl C. Sutherland,[16] Members, ASCE, of

[14] "Stresses in Concrete Pavements Computed by Theoretical Analysis," by H. M. Westergaard, *Public Roads*, April, 1926, p. 32, Eq. 12.

[15] "Analytical Tools for Judging Results of Structural Tests of Concrete Pavements," by H. M. Westergaard, *ibid.*, December, 1933, p. 185, Eq. 5.

[16] "The Structural Design of Concrete Pavements," by L. W. Teller and Earl C. Sutherland, *ibid.*, October, November, and December, 1935; September and October, 1936; and April–May–June, 1943.

the Division of Tests, of the Public Roads Administration, should be mentioned especially. The last part of this report contains extensive and careful comparisons of tests and theory. These include a very satisfactory verification of the relation defined by Eq. 7 between the stresses in the directions of the long and short axes of an oblong footprint of a tire.[17] The new formulas that have been published are derived from the same basic theory as the formulas published previously, and may thus be considered to be supported by the same experimental evidence.

The reasonableness (in the light of general experience) of the sequence of numerical values of stresses and deflections obtained in the six numerical examples may be considered as further evidence in favor of the formulas.

Conclusions

The formulas that have been presented lend themselves to three purposes: The interpretation of experimental evidence; the interpretation of practical experience with pavements in service; and the forming of judgments from which policies of design may be adopted to meet new conditions such as new types and new magnitudes of the loads.

In relation to the complexity of the problem, which involves a number of significant variables, the formulas are simple to use.

Acknowledgments

The manuscript from which this paper was prepared was the result of studies made by the writer while serving on active duty at the Bureau of Yards and Docks, United States Navy Department, as Commander, Civil Engineer Corps, United States Naval Reserve. Throughout the studies their use by the Board of Consultants on Airfield Pavements, Office of the Chief of Engineers, United States War Department, was kept in mind. The opinions contained in this paper are the private ones of the writer and are not to be construed as official, or as reflecting the views of the Navy Department or of the Naval Service at large.

The writer is especially indebted to the Staff of the Mathematical Section, Technical Service, of the Navy's David Taylor Model Basin, at Carderock, for supplying the numerical computations of the constants in Table 1.

[17] *Transactions*, ASCE, Vol. 108, 1943, pp. 864–868.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DESIGN OF PLYWOOD I-BEAMS

Discussion

BY HOWARD J. HANSEN

HOWARD J. HANSEN,[19] Assoc. M. ASCE.[19a]—The encouraging comments of those who discussed the paper seem to indicate that plywood I-beams have a definite place in the future of structural engineering. However, considerable test data and analyses are still needed. This is true not only of the I-beam but also of other types of cross sections.

As stated by the late Mr. Hogue,[19b] the anisotropic character of wood makes any analysis complicated; but, by expressing complicated equations in the form of simplified charts and tables, convenient design factors are established. With the advances made in timber and plywood analyses in the past few years, the need for the simplification of design data is apparent.

The writer did not plan his paper to cover all the possible conditions, and the limitations of the methods presented were noted in the "Synopsis." Mr. Ebeling pointed out several factors which are known by most engineers familiar with timber and plywood design. However, for those who are unacquainted with the Forest Products Laboratory (Madison, Wis.) formula for the most efficient section with unbalanced flanges and the effects of placing the face grain of the plywood web at an angle of $\pm 45°$ to the span, his comments are timely.

Mr. Oesterblom's comments concerning the future of plywood are in accord with the writer's views. His suggestions as to possible girder cross sections seem feasible. It is the writer's observation that the box beam with two or more webs probably has more possibilities than any other cross section. The same design methods can be extended to this type of section with little variation.

The extension of the writer's method by Mr. Novick and the inclusion of the curves shown in Figs. 5 and 6 are appreciated. They have already saved considerable time and effort. The paper was originally written in 1944 and

NOTE.—This paper by Howard J. Hansen was published in June, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1946, by Dick W. Ebeling, I. Oesterblom, and C. J. Hogue; and January, 1947, by Sidney Novick.

[19] Associate Prof., Civ. Eng. Dept., Univ. of Florida, Gainesville, Fla.

[19a] Received April 2, 1947.

[19b] Mr. Hogue died in November, 1946.

edited while the writer was overseas; consequently, access to the publications mentioned by Mr. Novick was impossible. The writer was pleased to note that the $\frac{d_w}{b_w}$-values checked by a close margin. This also was noted by Mr. Ebeling in his statement that the writer's theoretical method of spacing stiffeners agreed closely with that recommended by the Douglas Fir Plywood Association as checked by tests.

This paper should not be closed without mentioning a few of the many unsolved problems concerning wood and plywood when used in this or similar types of structures. Considerable information is needed on nailed joints. The paper considers flanges glued to the plywood web, but there is no reason why these cannot be nailed when sufficient test data are available on the spacing of nails. Since the flanges of an I-beam or box beam may be built up of several pieces nailed together, more information is needed on the size and quantity of nails to use. There is no question that glued laminated flanges fastened to the plywood web are more efficient than nailed flanges, but just how much more efficient is not known at present. Nailed girders have been used extensively in Europe, and, in place of plywood, diagonal 1-in. boards have been used for the web. This seems to present an interesting possibility, and tests should be made of girders of this type with American species. Complete frames or bents of this type of construction or with a plywood web should be analyzed and tested. Additional data are also needed on the use of plywood for gusset plates in timber trusses and arches.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SOME THOUGHTS ON ENGINEERING EDUCATION

Discussion

BY HENRY B. LYNCH, AND ALFRED R. GOLZÉ

HENRY B. LYNCH,[44] M. ASCE.[44a]—A well-considered, thoughtful statement of a problem of greatest importance to the engineering profession has been presented by Mr. Baker. In the past few years it has become increasingly necessary that the time devoted to preparation for an engineering career be scheduled with precision. More subjects should not be crowded into the college curriculum. In fact, there is criticism in some places that too much is attempted already. It has become necessary either to extend the time devoted to educating the engineer or to jettison some of the material formerly taught, to make room for new material that cannot be omitted. The author has suggested a good working basis for extending courses.

In appraising engineering education, in listing the subjects that can be discarded or curtailed, and in naming courses that could be given greater weight, the views and experiences of mature practicing engineers throw as much light as do those of engineering educators.

For many years it has seemed to the writer that the most important duty imposed upon the engineering college is that, first of all, the graduate be a broadly educated man. The aim of the curriculum should have been to impart judgment rather than skill, knowledge rather than training. He should have a broad knowledge—even though not a detailed knowledge—of history, English, and literature. He should be well grounded in mathematics and science and such foreign language as seems proper, without forgetting that he is being educated to become an engineer.

These objectives cannot all be accomplished unless there is a sharp curtailment in the time now devoted to subprofessional training. In an education leading to an engineering degree, much of the shopwork should be eliminated

NOTE.—This paper by Donald M. Baker was published in April, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1946, by E. S. Boalich, Russell C. Brinker, I. Oesterblom, Samuel T. Carpenter, Lynn Perry, and L. E. Grinter; October, 1946, by Robert O. Thomas, Clement C. Williams, N. W. Dougherty, H. A. Wagner, M. E. McIver, and Scott B. Lilly; December, 1946, by William A. Conwell, and A. Amirikian; and February, 1947, by C. A. Dykstra, and Louis Balog.

[44] Cons. Engr., Los Angeles, Calif.

[44a] Received August 27, 1946.

and such courses as surveying and mechanical drawing should be greatly shortened. Even so, provision should be made in some such manner as that proposed by the author, for extension courses preparing for further degrees.

These objectives cannot be attained through the college alone. Considerable groundwork must be done in the preparatory schools. For this purpose a much sharper distinction should be made between education that ends in the high schools and that which leads to a college degree. In preparing the engineering student, the high schools could well reduce greatly the elective subjects permitted, with a view to giving a thorough grounding in those subjects which a broadly educated person must have, so that the colleges may devote more time to the necessary engineering subjects.

ALFRED R. GOLZÉ,[45] ASSOC. M. ASCE.[45a]—The experience of the Bureau of Reclamation shows a need for greater appreciation by the engineering schools of the fields of endeavor into which graduates will advance. This experience supports Mr. Baker's conclusions that engineering education should emphasize fundamental principles in subjects throughout the broad field of engineering and that a graduate engineer should be able to undertake a career leading to administrative, executive, or managerial responsibilities. The unusual conditions affecting educational institutions during and following World War II provide an unexcelled opportunity for possible curriculum adjustments to meet extreme weaknesses.

The Bureau of Reclamation is considered primarily an engineering organization. Its accomplishments in the fields of civil, electrical, and mechanical engineering are well known; yet of its present staff of approximately 15,000 employees less than 30% are engineers devoting their time exclusively to strictly engineering phases of planning, design, and operation processes. The other employees—legal, clerical, and skilled labor—are engaged in multitudinous tasks of administration. Since the work of planning, building, and operating reclamation projects is essentially engineering in character, it follows that men with engineering training would seem to be the most logical group to advance to administrative and supervisory positions. It has been the Bureau's experience, however, that engineers, regardless of the excellent technical education they receive in the colleges, are not as well equipped to move into supervisory and administrative positions where employees of all professions and trades come under their direction, as are men who have received their basic education in fields with broader perspectives. For instance, men in charge of operating reclamation projects must be versatile creatures. They must excel at public relations in dealing with both water users and their innumerable requests, and with labor, both organized and unorganized. They must be good accountants to recover the construction and operating costs assessed against a project; they must write an endless number of brief, concise reports of every description; and they must operate and maintain massive dams and miles of canals, laterals, and drains, and direct the operation of fleets of equipment.

---

[45] Asst. Director, Branch of Operation and Maintenance, Bureau of Reclamation, Washington, D. C.

[45a] Received March 24, 1947.

The Branch of Operation and Maintenance of the Bureau of Reclamation has the responsibility for the operation of completed irrigation facilities. There is a woeful lack of qualified men to fill developing vacancies in the field. On the older irrigation projects the officials in charge are reaching retirement age. New projects requiring complete staffs of competent men are being brought to completion. In both cases the Bureau has no source of trained material from which to draw to fill the key positions on operating projects. This condition is caused not only by the failure of the Bureau to appreciate its own needs and to provide a proper training program, but also by a feeling on the part of most men who elect to stay with the civil engineering profession that the field of construction is the one to follow.

With the college curriculums in civil engineering shaped as they are in most schools, it is only natural that the graduate should judge that the sole professional opportunities are in construction. The lack of interest in the operation of completed engineering facilities—which is only one example—seems to be due to two factors: (1) A failure on the graduate's part to appreciate the engineering importance of these facilities to the continued welfare of the community and of the United States; and (2) the mercenary aspects of low salary scales.

With respect to factor (1), if Mr. Baker's engineering course, alternative I, were adopted, it would be of great assistance. In regard to factor (2), the Bureau of Reclamation, together with other federal agencies, pays the same salaries for operation and maintenance duties as those given to construction and planning engineers, largely eliminating the salary differential as a reason for keeping young engineers out of this field.

Seeking to determine the situation in the western universities, the writer has recently completed a trip through California and the Southwest where he visited some of the major universities giving courses in engineering and irrigation agriculture. In nearly every case, although the faculties appreciated the need for emphasizing the part that engineers can play in western development, the colleges were making no particular effort in that direction. In other words, the standard four-year engineering courses are about the same as the courses given in the universities of the Middle West and the East. Most colleges consider four years academically too short a time to provide an engineering student with other than the technical tools with which to begin his career. Such activities as public speaking, administration, western resource development, and national economics are seldom considered.

Resorting again to the Bureau of Reclamation as an illustration, it has expended more than $1,000,000,000 in providing a high standard of living to secure agriculture for the people of the West. It has on its books nearly another billion dollars of authorized work and ahead are more years of billion dollar expenditures. In thirty years the men who are now in colleges will be in charge of this vast investment. It is self-evident that the universities, particularly those of the West, carry a major responsibility in properly training men for this difficult and important work.

A promising aspect of the situation at most western schools is the evident desire of the faculty members to widen the scope of the engineering training and to increase the consciousness of students to their future responsibilities in

developing and administering the natural resources of their part of the United States. Shortcomings in public relations and report writing are well understood. Formerly several of the western schools were leaders in these fields and their staff members have become nationally known for their contributions to irrigation engineering. With the passage of years, however, there has been a slowing down of the leadership because of age and turnover so that institutions which formerly led the field are having difficulty in maintaining their rank. If other schools were coming to the front this would be a healthy development injecting new blood and new ideas in the field of engineering education, but there is little indication that a broader engineering education adapted to the needs of the consumer is becoming available.

Although the situation appears gloomy, the spark of self-improvement does exist in most of the schools visited by the writer, and presumably it exists elsewhere. Once the peak of the present high enrolment of returning veterans has been passed, a number of western schools should seek to motivate engineering curriculums to reflect the needs of the nation, and particularly those of the western states, by arousing in the neophyte engineer a consciousness of the responsibilities attending his profession in the broad fields of economics, public relations, and administration. If the users of engineers, as well as the producers of engineers, will accept Mr. Baker's challenge, this objective ultimately should be forthcoming.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CORRECTION OF TAILWATER EROSION AT PRAIRIE DU SAC DAM

Discussion

BY GEORGE P. STEINMETZ, LOUIS M. LAUSHEY, ABRAHAM STREIFF, AND R. H. KEAYS

GEORGE P. STEINMETZ,[6] M. ASCE.[6a]—The correction of tailwater erosion at the Prairie du Sac Dam (Wisconsin) by the construction of an additional apron and baffles was well planned, thoroughly investigated, and has proved to be highly successful. This project again emphasizes the economic advisability of careful model testing to determine the most advantageous corrective methods, as well as the nature of operation to be followed in making maximum use of corrective facilities after they have been installed. The erosion, necessitating corrective measures, further emphasizes the economy and desirability of model testing before initial construction of new projects when the behavior of foundation material cannot be anticipated with certainty. It is appreciated, however, that at the time this dam was constructed the science of model testing of this nature was not well developed.

The increase in bottom velocities in run No. 27 over run No. 26, with tailwater below normal in the latter run, as shown in Table 3, indicates the advisability of a gradual building up of tailwater elevation to normal before the gates are opened to the full extent necessary to pass a given flood so that excessive bottom velocities may be avoided.

The effects of flow distribution, as it might be controlled by particular combinations of gate openings, are indicated in the first paragraph of the "Conclusions." The determination and adoption of the most advantageous combinations of gate opening for various rates of flood and power generation discharges may be particularly important.

The entire problem of controlling tailwater erosion at the Prairie du Sac Dam resulted from the characteristics of the shifting sand bed of this stretch of the Wisconsin River. Some understanding of these conditions can be obtained by reading the 1913 and 1916 articles concerning the construction of the dam

NOTE.—This paper by C. N. Ward and Henry J. Hunt was published in October, 1946, *Proceedings*.

[6] Chf. Engr., Public Service Comm., Madison, Wis.

[6a] Received February 21, 1947.

which were cited in the paper.[3,4,5] In this section of the river, where the sand bed is not controlled by a dam or other barrier, the bed is in a continual process of moving downstream in the form of moving sand bars. This movement is particularly evident at the stream gaging station at Muscoda, Wis., on the Wisconsin River, a number of miles downstream from the dam. At this gaging station, which was established at about the same time as the Prairie du Sac Dam was constructed, it is a common occurrence to observe a sand bar upstream from the gaging station during the early part of the open water season. As the season progresses, the bar moves closer to the station, finally reaching it; and at that time a normal depth of water of from 8 ft to 9 ft at the gaging station is replaced by 6 in. to 1 ft of water over the surface of the bar.

Soundings at the Muscoda Bridge for regular discharge measurements during the period 1914-1929 give some indication of the amplitude of moving sand bars. At eleven representative stations across the channel the extreme changes in bottom position were from 4 ft to 22 ft in 15 years, with an average range of nearly 8 ft for the eleven stations. Airplane photographs of the river show the sand bars quite clearly. No reliable information exists, however, on the speed with which these bars pass downstream, or the bed-load volume involved. Historically, this movement of sand bars has been in progress since early log rafting days on the river, but it appears that the downstream movement had not been seriously considered. When a large pond is created by a dam on the river, the drifting bars above the dam are arrested in the pond, but downstream from the dam the movement continues, thereby lowering the stream bed and the tailwater at the dam. Immediately downstream from the dam the movement of material is greatly accelerated by the new and rapid currents introduced by the dam.

It appears that until such time as it may become economically feasible to develop slack water throughout the sandy reach of this river, about 200 miles in length, any dam constructed on the sand bed will experience a lowering of the stream bed similar to that experienced at Prairie du Sac, and, therefore, measures to retard or to take advantage of such erosion will be advisable. It is doubtful if such retardation can be effected without the creation of a deep pool or stilling basin, such as was rapidly created by nature at the Prairie du Sac Dam through the erosive action of flood discharges.

Perhaps the initial construction of an apron and baffle system on any future projects, such as has been developed at the Prairie du Sac Dam, over a period of time, would result in the creation of a relatively stable stilling basin in the sand bed by the natural action of floodwater discharge.

In the event that a project was designed, as indicated herein, to create a stilling basin or pool through designed erosion, it would be advisable to determine by model tests the extent of erosion in different periods of time and so to design draft tube outlets, locks (if any), and other parts of the structure for the ultimate tailwater elevation and head that would be created.

---

[3] "Construction of the Prairie du Sac Power Plant," *Engineering Record*, May 31, 1913, p. 603.

[4] "Method of Constructing a Hydro-Electric Power House and Dam on Sand Foundation," *Engineering and Contracting*, May 7, 1913, p. 533.

[5] "Building Power House and Dam on Sand Foundation," *Engineering News*, June 15, 1916, p. 1113.

The piling foundation of the apron addition, which is designed to withstand an uplift of about 13 ft of head, is unique in Wisconsin. It is particularly important in this project, in that it allowed the effective head on the original structure to be decreased and, at the same time, allowed the total head on the project to be increased. The result was that the output was increased, particularly during periods of low stream flow when tailwater is lowest and firm power most essential.

LOUIS M. LAUSHEY,[7] JUN. ASCE.[7a]—Different methods have frequently been used to dissipate the kinetic energy possessed by the high-velocity jet on the apron, the best of which has probably been the use of a hydraulic jump. The problem at Prairie du Sac was a gradual lowering of the tailwater to such a depth that apparently a jump would not form on the apron for all discharges.

If the discharge and initial depth are known, the downstream depth required for the jump to form can be computed from the momentum equation. When the tailwater depth is insufficient, a jump will not form on the apron. The authors have used the common method of interposing a baffle or sill in the path of the jet to create an additional upstream force to aid in the formation of the jump at the desired position. The obstructions also have the additional value of deflecting the high-velocity filaments toward the surface, the result being to keep the velocities on the bottom at a minimum. Frequently, a roller can be produced that actually causes the bottom velocities downstream from the end sill to travel toward the dam and to build up any scoured material behind the sill.

Tests are being made at Carnegie Institute of Technology in Pittsburgh, Pa., to determine the effectiveness of a rectangular end sill on the production of a hydraulic jump when the tailwater depths are below normal. Some experiments have been made in a rectangular channel 12 in. wide and 30 ft long. A thin, high-velocity jet was admitted to the upstream end by confining the discharge between the channel bottom and a low, rigid plate which extended across the channel and was parallel to the bottom. The inlet plate was adjusted to different depths in such a manner that the water surface upon exit into the channel was parallel to the bottom. The jump was formed at the lower end of the channel after traveling about 20 ft. It was assumed that a velocity distribution would be produced at the beginning of the jump closely resembling that of the discharge after flowing down the spillway of a dam. The initial and final depths, and the discharge, were measured when a jump was formed on the apron without a sill, and with sills of various heights. Tailwater depths were measured with piezometers at 2-ft intervals for a distance of 6 ft downstream from the sills. An adjustable gate at the lower end of the channel made it possible to adjust the tailwater to obtain the minimum depth required to form a jump. Minimum depth was defined as the lowest depth for which the piezometers downstream from the sills recorded the same height. This setup insured that the depth recorded in the model would be comparable to the actual depth in the prototype rating curve. Furthermore, any depth below this minimum caused a wavy surface that was highest over the sill, and the possibility

[7] Instructor in Civ. Eng., Carnegie Inst. of Tech., Pittsburgh, Pa.

[7a] Received February 28, 1947.

of high velocities being deflected toward the bottom was considered more likely than when the depth downstream was constant. The use of a second baffle, as at Prairie du Sac, would decrease the piling up over a single baffle with a resulting drop in the water surface to the actual tailwater elevation.

Analysis of Fig. 3(*b*), given for the 1929 tests by the authors, shows that at low discharges a jump does not form to increase the depth below the sill. Actually the baffle for the smaller flows, when the tailwater is very low, acts as an overflow weir. As a result the water drops below critical depth on the apron downstream from the baffle to nearly its original depth. This condition holds true to a lesser extent for the higher flows. Any increase of tailwater depth to the height over the sill would not cause the jump to move upstream. The fact that a small amount of scouring resulted during operation of the spillway is probably a result of the small velocities at low flows, and of the higher depths and the increased effectiveness of the baffle to form a jump at the larger discharges. An added protection against scouring was obtained by placing the baffle 20 ft from the end of the apron. The "drop-off" from the end of the apron to the unpaved stream bed, obtained in the 1929 tests, would aid in giving good results by reducing the velocities because of the added depth of flow.

Perhaps of most importance in the correction of the erosion was the excellent velocity distribution obtained by the authors as shown in Fig. 5 where the bottom velocities either were zero or were directed upstream when the new apron was installed. This is a necessary function of any sill or baffle— the formation of a low bed velocity. The effect that such a device has on dissipating energy and forming a jump at lower tailwaters than otherwise required is of little importance if high velocities reach the bottom.

From experimental curves and work in process at Carnegie Institute of Technology, an attempt has been made to analyze the height of baffle of 52 in. used in the 1929 tests for forming a jump with low tailwater depths. The initial jet depth for 50,000 cu ft per sec (50.8 cu ft per sec per ft of apron), from Fig. 3(*b*), scales between 1.5 ft and 2 ft. For the former initial depth, it would be possible to produce a jump, with no drop in elevation anywhere downstream, with an actual stream depth only 80% of that required without a baffle. Using the other limit of the range would require a depth of 87% of normal. The ordinary hydraulic jump formula would require a tailwater depth at the baffle of between 8 ft and 9.5 ft, depending on the assumed initial depth. Thus, the required elevation of tailwater for 50,000 cu ft per sec with no baffle is between 751 ft and 752.5 ft. Fig. 3(*b*) shows an actual depth of 749 ft or a depth, without a baffle, at 20 ft, of 6 ft of water. This yields a percentage ratio, at the distance −20 ft, of depth available to depth required for a jump on the apron of between 63% to 75%. Therefore, it seems that the tailwater depth is about 15% too low for a jump with no loss in depth to form at this discharge. That this estimate is perhaps reasonable can be seen by inspection of the profile in Fig. 3(*b*) where the height over the baffle is about 6 ft above the stream elevation. Experiments indicate that at this flow a baffle less than two thirds of the height used would have produced a jump at 50,000 cu ft per sec. Of course, the baffle would have to be designed for 65,000 cu ft per sec, and this flow would require a higher baffle. The maximum discharge seems to be drowned out on

the bucket, and the depth at distance −42 ft from the end of the apron seems to be high for the jet depth. Therefore, it is unfortunate that a similar analysis could not be made for this discharge. At any rate, the amount of scouring is of most importance, and undoubtedly the 56-in. baffle was used on the basis of best protection from scour, and not just on the formation of a hydraulic jump.

ABRAHAM STREIFF,[8] M. ASCE.[8a]—An important contribution toward the solution of several questions involved in the design of dams is contained in this paper. In particular, the publication of the 33-year experience record of the Prairie du Sac Dam is a valuable addition to the literature on scour below dams. This dam, on a sand foundation, designed by Daniel W. Mead, Hon. M. and Past-President, ASCE, is a remarkable structure in many respects. The authors describe mainly the conditions and corrections of tailwater erosion, which will be discussed first. Several other features are also mentioned which warrant special discussion.

The main and important conclusion to be drawn from the paper is the following: To be considered successful, a dam structure should be permanent and secure. To this end the paper demonstrates that it is unnecessary to design against the occurrence of all erosion below the apron and that the aim of the designer should be to prevent such erosion. If the design is such that the erosion itself is stable, and does not endanger the structure, the design may be considered completely successful. Experience has taught how to attain this end.

There are many dams with erosion below the apron, not only on friable soil foundations, but also on the hardest rocks, such as gneiss.[9] In many of these cases corrective measures have been taken; but in others, with equally successful results, no such measures were necessary. These structures, built since the late 1880's, all illustrate the truth of the foregoing statement. An important additional proof has been given by the paper.

This principle should be emphasized because recent discussions frequently center around the building of aprons so that: (1) Erosion downstream is prevented; (2) the hydraulic jump is created; and (3) erosion occurs on the apron.

(1) *Prevention of Downstream Erosion.*[10]—This requirement is frequently advanced, but the necessity of preventive measures is not sustained by actual experience, nor is it followed in actual design. The Prairie du Sac Dam in Wisconsin is only one of many examples. It is important because the maximum erosion, of one and one-half times the normal head, occurred in a sand foundation. The dam is apparently entirely secure.

The width of the structure—104 ft from upstream sheet piling to downstream end of apron—is very short considering the head of 32 ft and the sand foundation. Comparison with the structures, listed in Table 4(*a*) built during the same period and on similar foundation material, reveals that they have aprons much longer than the apron of the original Prairie du Sac Dam. Hence,

[8] Fargo Eng. Co., Jackson, Mich.

[8a] Received March 20, 1947.

[9] "Technisch wirtschaftliche Fortschritte auf dem Gebiete der Wasserkraftanlagen in der Schweiz," by E. Meyer-Peter, World Power Conference, Vol. IX, 1930, p. 77.

[10] "Development and Hydraulic Design, Saint Anthony Falls Stilling Basin," by Fred W. Blaisdell, *Proceedings*, ASCE, February, 1947, p. 123.

none of these dams shows any erosion below the apron. Although this condition is satisfactory, the question arose as to whether a considerable saving in cost might not be attainable by making shorter aprons and by allowing a certain amount of stable erosion, especially since a number of existing and older structures show that this result is quite feasible as has been proved again by the paper. The original apron of the Prairie du Sac Dam was lengthened 50 ft. The baffle directs the flow upward, and thus corrects the original downward inclination of the apron. The same effect is also obtainable by a toe weir with inclined upstream face. A number of dams built later than the Prairie du Sac Dam, on soil foundations, have shorter aprons and have permanent but harmless erosions below the structure (see Table 4(*b*)). These dams have experienced maximum flood discharges of 115,000 cu ft per sec several times, and have permanent but harmless downstream erosions.

TABLE 4.—OVER-ALL UPSTREAM AND DOWNSTREAM LENGTH OF EXISTING DAMS ON SOIL FOUNDATIONS

Dam	Location	Head (ft)	Year built	Length [a] (ft)
	(*a*) LONG APRONS			
Croton	Muskegon River, Michigan	40	1907	289.75
Mio	Ausable River, Michigan	32	1915	245.21
Alcona	Ausable River, Michigan	40	1922	253
Junction	Manistee River, Michigan	55	1917	292.75
	(*b*) SHORTER APRONS			
Prairie du Sac	Prairie du Sac, Wisconsin	32	1914	104
Dunlap	Guadalupe River, Texas	35.2	...[b]	122
Abbott	Guadalupe River, Texas	31.0	...[b]	110
Nolte	Guadalupe River, Texas	24.7	...[b]	93
Texas Power Company: IV	Guadalupe River, Texas	30.5	...[b]	112
Texas Hydro-electric Company, H4	Guadalupe River, Texas	28	...[b]	118
Texas Hydro-electric Company, H5	Guadalupe River, Texas	29	...[b]	118

[a] Length from upstream sheet piling to end of apron. [b] Various years between 1927 and 1931.

There are much older structures on soil foundations which have permanent and harmless deep erosions downstream. An example is the Trowbridge Dam on the Kalamazoo River, in Michigan, which was built in the 1890's on a silt and sand foundation. The head is 22 ft and a permanent erosion 40 ft deep exists downstream from the pole apron.

Many dams with permanent downstream erosions also exist on rock foundations.[11,12] Other examples, in Europe and in the United States, are numerous.[13,14,15] The only requirements are that the erosions be stable and that they not tend to undermine the structure. Considerable experience is recorded to assist in determining how such results can be attained. The paper clearly states this important conclusion.

[11] "Concentrated Flow Erodes Rock Below Wilson Dam," by Hugh P. Oram, *Engineering News-Record*, February 3, 1927, p. 190.

[12] "The O'Shaughnessy Dam and Reservoir," by John H. Gregory, C. B. Hoover, and C. B. Cornell, *Transactions*, ASCE, Vol. 93, 1929, pp. 1501–1502.

[13] "Kolk abwehr," by A. Schoklitsch, Julius Springer, Vienna, 1935.

[14] "Mécanisme de l'Eau," by René Koechlin, Béranger, Paris, 1926.

[15] "Reservoirs," by James Dix Schuyler, John Wiley & Sons, Inc., New York, N. Y., 1908.

(2) *Creation of a Hydraulic Jump.*—At low heads and at discharges of great depth, the hydraulic jump is an imperfect method of destroying energy, as may be noted from available tables of hydraulic jump data.[16] Therefore, there is no merit in attempting (as so often is done) to create a sufficient depth on the apron to cause the formation of a hydraulic jump at low heads. The authors illustrate this fact clearly in Fig. 6(*b*) where, although the upper baffle is too high to create a true jump, it, nevertheless, proved effective.

(3) *Locating the Jump on the Apron.*—In low-head design, this often-specified requirement has been proved unnecessary. It is more important to direct the nappe toward the surface, which the authors achieve by baffles, as stated under "Conclusions." This procedure creates as much room as possible for the upstream bottom velocities of the "ground whirl." The erosion caused by the extension of the turbulence below the apron, as the authors show, is permanent and harmless, even though the erosion depth is 20 ft below the new apron and the foundation consists of sand. Experience with many other structures, some of which are listed in Table 4, illustrates this fact.

(4) *Lowering of Tailwater Elevation.*—The lowering of tailwater is a familiar phenomenon on many rivers in alluvial beds. The dam intercepts the movement of bed load, and below the dam the existing grade creates new bed load and degrades the tailwater. The total for the Prairie du Sac Dam was fortunately not great; apparently it was not more than 6 ft after which it seems to have been stabilized. Greater increases have occurred in some Michigan dams. At the Junction Dam on the Manistee River the tailwater was lowered 10 ft and was only stabilized by the construction of a weir downstream, which is only temporary. The degradation continues. In 1917, when the dam was designed, previous experience with the Croton Dam on the Muskegon River caused the writer to place the Junction draft tubes 10 ft lower than necessary. Even this amount proved insufficient, for the draft tubes have been corrected since, and later a counter weir was built to stabilize the head. The apron, which was previously submerged, is "high and dry" at low water. This lowering of tailwater is much in evidence in the southwestern United States—for instance, at Boulder Dam.

(5) *Other Features of Prairie du Sac Dam.*—Several other interesting and successful features of the Prairie du Sac Dam are shown in the paper. Since there is no base slab, the upstream sheet piling forms the principal cutoff. There is no horizontal percolation distance; the reduction in head by the sheet piling is 27 ft or 75% of the present head. At the Mio Dam, which has a base slab, the upstream piling reduces the head by 50%. The advantage of the design shown in Fig. 1 is obvious and apparently has been quite successful.

The dam has forty-one Tainter gates 20 ft wide by 14 ft high. The dam is on a large river in an area that has a severe winter climate. Apparently a multiplicity of rather small gates has not been a handicap under severe ice conditions. It would be of great interest to learn from the authors how the spillway is guarded against ice thrust and is kept operative during the winter and during the breakup of the ice.

[16] "Hydraulics of Open Channels," by Boris Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 247.

R. H. KEAYS,[17] M. ASCE.[17a]—The project described by the authors is very interesting to the writer, as several years ago he had a similar problem in connection with the flood control works on the Struma River in Macedonia, Greece.

The term "tailwater erosion" is not strictly accurate; the authors would do better to state that the erosion was caused by the turbulent flow from the dam. It is understood that the tailwater elevation is measured after the turbulent area is passed and the river has assumed a normal flow.

The "Synopsis" states that in the course of time there was a general lowering in the tailwater level and the river bottom downstream from the dam. It had been expected that long since, another dam would have been constructed downstream, which, presumably, would have backed up the water enough to

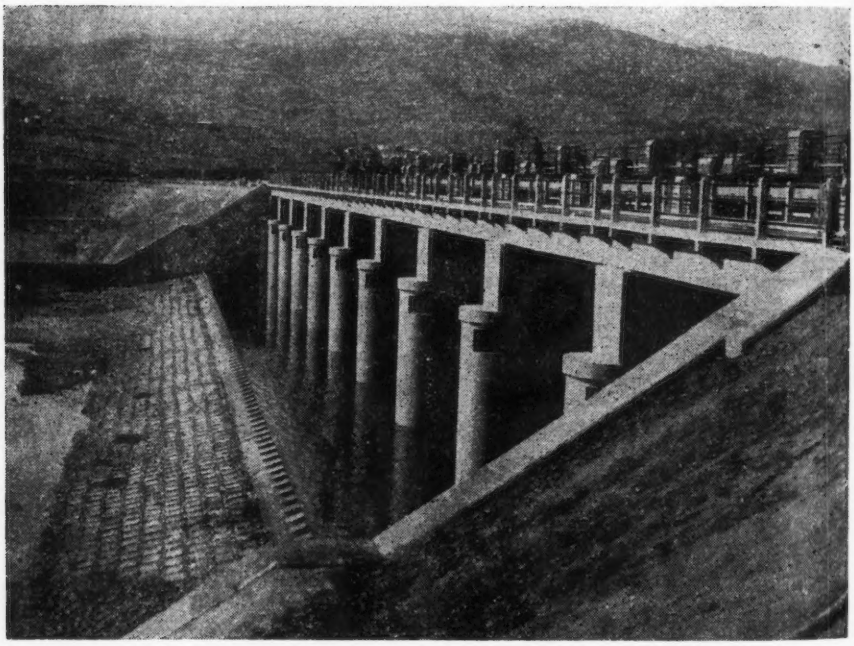


FIG. 7.—DOWNSTREAM VIEW OF WEIR, SHOWING ARTICULATED BLOCK PAVING AND DENTATED SILL

maintain the tailwater elevation and prevent erosion of the river bed. As this was not done, however, erosion continued and a sharp drop developed at the Prairie du Sac Dam, in Wisconsin. It would be interesting, therefore, to know how much erosion has occurred at a distance from the dam sufficient to be free from the influence of the turbulent flow.

The Kerkini weir on the Struma River near the north end of the Struma Plain is a structure similar to the Prairie du Sac Dam (see Fig. 7). It also is built on a floating foundation in sand with a line of steel sheet piling at the upstream and downstream ends. In all respects it is similar to the barrages on

[17] Jackson Heights, N. Y.

[17a] Received March 25, 1947.

the Nile River in Egypt, and was designed by an engineer with long experience in the Nile Irrigation Service.

The weir creates a large detention basin Kerkini Lake to regulate a flood of 4,000 cu m (141,000 cu ft) per sec, and this basin catches all the sediments brought down through the Rupel Gorge from the watershed in the mountains of Bulgaria to discharge into the Struma Plain.

The design of the weir was referred to Karl Rehbock at Karlsruhe in Germany, and his modifications of the design were based on hydraulic model experiments in his laboratory. Professor Rehbock's design included what he called a dentated sill. The dentations were a row of concrete "teeth" projecting upward at the downstream edge of the apron. They corresponded to the baffle wall described by the author. Professor Rehbock laid great stress on the importance of these dentations in breaking up the velocity of the current.

With this design it was calculated that a so-called "roll" would develop in the flow downstream from the apron. A roll can be described as a vertical eddy with the return flow at the bottom. The experiments showed that the erosion downstream from the apron would be in the form of a shallow depression extending for only a hundred feet or so.

Apparently, Professor Rehbock gave no consideration to the fact that the Struma River was a heavy sediment carrier. The result was that the relatively clear water discharge from the weir soon began to erode the bottom downstream from the weir for a long distance. This erosion was neutralized to some extent by material brought down by tributaries below the weir, but it was increased because of a cutoff having been made in the river some distance below the weir to shorten its course. This was a permanent lowering of the bed as no new sediment was coming down as a replacement.

This lowering of the bed, if carried far enough, would endanger the stability of the weir. As nearly as the writer can remember, the weir was designed for a flood flow of 1,200 cu m (42,400 cu ft) per sec. There was no reason to anticipate, however, that the weir would ever have to operate at more than 500 cu m (17,700 cu ft) per sec. It is reported that, at one time in 1935, it was probably operated by revolutionists at a rate considerably in excess of 500 m per sec.

The weir consists of a series of ten steel vertical lift gates supported by intermediate piers and wing walls at the ends, all supported on a concrete foundation on wood piles in a sand formation. To prevent seepage, lines of steel sheet piling are driven at the upstream and downstream ends.

The weir was well along to completion when the writer became chief engineer on the project in 1932. One of the first studies undertaken thereafter was to examine the question of the prospective erosion downstream. Even with a flow of 500 cu m per sec, or less, it was calculated that it would not be long before the stability of the weir would be in danger because of the difference in head allowing water to find its way under the steel sheet piling.

The weir was placed in operation in February, 1933. There was a small flow through the weir for some time—just enough to wash away some small sand deposits above the weir and deposit them on the flatter slope below the weir. As soon as these deposits were exhausted, however, erosion began downstream all along the course of the lower river.

There is no reliable information available as to the present condition of the Struma improvements. It is reported that they were largely destroyed during World War II. The upper works of the Kerkini weir certainly were destroyed as evidenced by photographs in possession of the writer.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SPACE RESECTION PROBLEMS IN PHOTOGRAMMETRY

Discussion

BY GEORGE H. DELL

G. H. DELL,[9] Assoc. M. ASCE.[9a]—The graphical methods in this paper, supplemented in places by numerical calculations, provide a comparatively rapid determination of the required orientation data. The accuracy of the solution should prove adequate in most instances, particularly with the refinements afforded by Eqs. 29 and 28a. The author's adaptation of the theory of the line of position is noteworthy as a step in extending this concept to new fields.

The writer has found that the angles 1 to 6, Fig. 3, can be obtained fairly rapidly, to the nearest second, by the use of six-place log sines (see Eqs. 9). For the numerical example given by the author the equations are as follows:

$$\log \sin \angle(5) = \log \sin \angle(2) + 31{,}259 \quad \ldots\ldots (37a)$$

$$\angle(6) = 139°07'18'' - \angle(5) \quad \ldots\ldots (38a)$$

$$\log \sin \angle(3) = \log \sin \angle(6) - 10{,}350 \quad \ldots\ldots (37b)$$

$$\angle(4) = 146°36'35'' - \angle(3) \quad \ldots\ldots (38b)$$

$$\log \sin \angle(1) = \log \sin \angle(4) - 20{,}909 \quad \ldots\ldots (37c)$$

$$\angle(2) = 131°03'15'' - \angle(1) \quad \ldots\ldots (38c)$$

TABLE 7.—DETERMINATION OF

Number	First Cycle		Second Cycle		Third Cycle		Fourth Cycle	
	Value	Log sin	Value	Log sin	Value	Log sin	Value	Log sin
2.....	62°00′	9.94593	61°09′	9.94245	60°58′	9.94168	60°55′30″	9.941504
5.....	71 35	(9.97719)	70 16	(9.97371)	69 59	(9.97294)	69 55 10	(9.972763)
6.....	(67 32)	9.96572	(68 51)	9.96971	(69 08)	9.97054	(69 12 08)	9.970737
3.....	64 28	(9.95537)	65 36	(9.95936)	65 50	(9.96019)	65 52 55	(9.960357)
4.....	(82 09)	9.99591	(81 01)	9.99464	(80 47)	9.99436	(80 43 40)	9.994288
1.....	70 45	(9.97500)	70 16	(9.97373)	70 10	(9.97345)	70 08 35	(9.973379)
2.....	(60 18)	....	(60 47)	....	(60 53)	....	(60 54 40)	....

NOTE.—This paper by P. H. Underwood was published in September, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1946, by Earl Church; and January, 1947, by Ralph O. Anderson.

[9] Asst. Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

[9a] Received April 7, 1947.

To begin with, an approximate value of $\angle(2)$ is used. This quantity may be found by the graphical method described by Professor Underwood (see Fig. 2), or may be obtained from the following equation, in which it is assumed that the photograph is without tilt:

$$\angle \text{LBA} = \angle \text{Lba} - \sin^{-1} \frac{\sin i_{BA} \sin \text{Lba}}{\cos m_b} \quad \ldots\ldots\ldots\ldots (39)$$

in which i_{BA} is the inclination of line BA with respect to the horizontal (positive for upward inclination). The values obtained by the writer for $\angle$LBA ($= \angle 2$) were: By the graphical method, 62°00′; and, by Eq. 39, 60°51′44″. The solution shown in Table 7, which verifies that presented by the author, begins with the former value. Quantities obtained from Eqs. 37 and Eqs. 38 are shown in parentheses in Table 7.

A method consisting primarily of plane trigonometry was used by the writer to check the calculation of the coordinates of L and V, and the results agreed within 0.1 ft, on the average, with those given by the author. Accordingly, in the calculation of i, α_{ov}, and α_{VO}, by a similar procedure, the author's data were used. Referring to Fig. 14, the vertical distances of points a, b, and c, with respect to point L—denoted by p_a, p_b, and p_c, respectively—were computed and the horizontal line be was located. Line fo is the trace of a vertical plane through point L perpendicular to line be. Point o is the principal point of the photograph, and point v is the vertical projection of point L on the horizontal plane through line be. Angle fbv is a horizontal

TABLE 8.—COMPUTATION OF TILT, SWING, AND AZIMUTH OF PRINCIPAL LINE

Symbol	Equations	Numerical results
p_a.....	$p_a = \overline{\text{La}} \cos M_A$	208.663
p_b.....	$p_b = \overline{\text{Lb}} \cos M_b$	208.859
p_c.....	$p_c = \overline{\text{Lc}} \cos M_c$	211.542
$\overline{ce}$.....	$\overline{ce} = \frac{p_c - p_b}{p_c - p_a} \overline{ac}$	147.107
$\angle$cbe...	$\tan \frac{\angle\text{cbe} - \angle\text{ceb}}{2} = \frac{\overline{ce} - \overline{cb}}{\overline{ce} + \overline{cb}} \tan \frac{\angle\text{cbe} + \angle\text{ceb}}{2}$	51°13′32″
i.....	$\sin i = \frac{\sin i_{bc}}{\sin \angle\text{cbe}}$	−1°26′46″
s.....	$s = \alpha_{bc} - (90° - \angle\text{cbe})$	225°59′00″
$\angle$obf...	$\angle\text{obf} = \angle\text{cbe} - (\alpha_{bo} - \alpha_{bc})$	11°26′12″
$\angle$fbv...	$\tan \angle\text{fbv} = \tan \angle\text{obf} \cos i - \frac{\overline{\text{Lo}} \sin i}{\overline{\text{ob}} \cos \angle\text{obf}}$	13°47′38″
α_{VO}....	$\alpha_{VO} = \alpha_{VB} - (90° - \angle\text{fbv})$	227°23′49″

BASE ANGLES

FIFTH CYCLE		SIXTH CYCLE		SEVENTH CYCLE		Number
Value	Log sin	Value	Log sin	Value	Log sin	
60°55′05″	475	60°55′14″	485	60°55′15″	9.941487	2
69 54 32	(734)	69 54 45	(744)	69 54 48	(9.972746)	5
(69 12 46)	768	(69 12 33)	757	(69 12 30)	9.970755	6
65 54 28	(418)	65 54 16	(407)	65 54 14	(9.960405)	3
(80 42 07)	256	(80 42 19)	260	(80 42 21)	9.994261	4
70 07 53	(347)	70 07 59	(351)	70 08 00	(9.973352)	1
(60 55 22)	....	(60 55 16)	....	(60 55 15)	....	2

angle, whereas angle fbo lies in the plane abc, the inclination of which is i_{fo} = the tilt, i. The equations and numerical results are given in Table 8, and again it will be seen that the author's solution is substantially verified.

In the course of these calculations it was noted that a relatively small error in the β-angles had a pronounced effect on the calculated coordinates of points L and V. The writer would like to inquire whether Professor Underwood has made an estimate of the probable error of these coordinates, in the example in question, assuming that the solution is without error other than in the determination of the coordinates of the control points on the photograph.

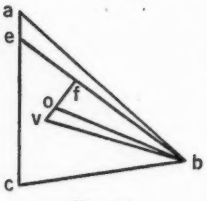


FIG. 14.

Attention is called to the fact that the computation of the angle λ, shown beneath Table 5, is accurate to only four significant places. Using the numerals given in brackets, the writer obtained 0.451024 as the value of the tangent, corresponding to an angle of 24°16′36″. The value obtained by the use of six-place logarithms was 24°14′53″, and, by seven-place natural functions, 24°16′19″. The solution in Table 8 involves a similar weakness, although to a lesser extent.

This paper deals with a very interesting problem, and the author has presented a commendably clear and well-written treatment.

Corrections for *Transactions:* In September, 1946, *Proceedings*, on page 953, in Eq. 28*a*, change "m_q" to "m_a"; and, on page 956, second line below Table 2, change "lines 7" to "lines 8."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CLEANING AND GROUTING OF LIMESTONE FOUNDATIONS, TENNESSEE VALLEY AUTHORITY

A SYMPOSIUM

Discussion

BY ROBERT A. LAURENCE

ROBERT A. LAURENCE,[9] ESQ.[9a]—Not only is the value of close, continuous cooperation between engineers and geologists emphasized in this Symposium, but also the changed and increased duties and responsibilities of the geologist since the late 1920's on such construction projects are illustrated. The changes result chiefly from the exhaustion of excellent sites and the emphasis on multipurpose projects. The discovery of ideal sites is becoming almost impossible; but, as the Douglas and Fort Loudoun examples show, detailed preliminary investigations and geologic study can result in the rejection of the most unsatisfactory sites, and a thorough knowledge of the subsurface conditions is necessary to insure proper treatment of the foundation finally selected. The Fort Loudoun site was selected after investigation of fourteen alternate sites, and the Douglas site was one of four originally considered. Furthermore, drilling for grouting the Douglas cofferdam revealed that, had the axis of the dam been located more than 100 ft upstream or downstream an additional 10 ft to 20 ft of highly cavernous rock would have been encountered in the spillway and powerhouse foundations.

These examples also illustrate the effect of seemingly insignificant lithologic or structural features in determining the course and extent of solution and weathering. The north abutment cave at Douglas Dam was caused by the presence of a shaly layer less than 1 in. thick, which had deflected the ground water along the 20° dip of the beds, thus concentrating the solution. Although (see Fig. 17) the cave was large enough for mining and backfilling only to Station 5 + 32 (El. 891), weathering and small cavities were present along

NOTE.—This Symposium was published in December, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1947, by Berlen C. Moneymaker, and C. E. Blee.

[9] Regional Geologist, U. S. Geological Survey, Jefferson City, Tenn.; formerly Geologist TVA, Douglas Dam, Tennessee.

[9a] Received March 24, 1947.

this bed as far as Station 13 + 0 (El. 679). The thin insoluble bed, which caused development of the south abutment cave for a total distance of more than 700 ft down the dip, was even less conspicuous.

The extent and continuity of cavities along bedding planes and strike joints are further shown by the fact that dye placed in the river, more than 1,000 ft upstream and downstream from the cofferdam, appeared in the same leak in the excavation area within from 30 min to 45 min.

No description of the Douglas foundation is complete without mention of the unusual filled sink on the south abutment between Stations 18 + 90 and 19 + 53. This sink was developed before the deposition of the Lenoir limestone, and was filled with 65 ft of extremely fine-grained volcanic ash and 55 ft of thin-bedded dolomite. It extended to El. 847; but, since the lower 40 ft of the volcanic ash was fresh, compact, and insoluble, excavation was "bottomed" at El. 881. Many core drill holes were put through the bottom of this filled sink but no outlet was found. Deep solution on the strike joint channel shown in Fig. 22 was closely related to this sink hole.

TABLE 5.—Results of Preliminary Core Drilling at Belle Canton Site, Fort Loudoun Dam

(Includes All Drilling Through October 31, 1939)

Rock type	Number of holes	Total in rock (ft)	Recovery (%)	Loss (%)	Cavity (%)
Tellico[a]	14	718.8	90.70	1.06	8.24
Tellico[b]	53	4,990.3	84.42	0.59	14.99
Sevier I[a]	12	658.5	86.71	4.02	9.27
Sevier I[b]	13	662.1	78.96	5.62	15.42
Sevier II[a]	42	2,225.8	85.58	10.70	3.72
Sevier III[a]	21	1,258.2	76.95	22.42	0.63
Total	155	10,513.7	(84.00)[c]	(5.91)[c]	(10.09)[c]

[a] Vertical holes. [b] Angle holes. [c] Averages.

Under the heading, "Geology," Mr. Pauls states that:

> "Contrary to the normal conception that relatively more cavities are to be expected in the purer limestone, the cavities in this area were primarily controlled by structural features rather than by the purity of the rock."

This statement is misleading. Although it is true that the shape, extent, and localization of the cavities were controlled by structural or sedimentary features, nevertheless all the major cavities at Fort Loudoun, as shown in Fig. 3, were in the Tellico, Sevier unit 1, and the lower part of Sevier unit 2. Although unit 3 and the upper section of unit 2 also suffered intense deformation, the rocks were much more shaly, and so suffered less from solution. The distribution of cavities in the various rock types is also demonstrated by Table 5, summarizing results of the exploratory drilling.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STABILITY OF SOIL SLOPES

Discussion

BY PAUL BAUMANN, AND GREGORY P. TSCHEBOTARIOFF

PAUL BAUMANN,[15] M. ASCE.[15a]—Research in the mechanics of slides in soil slopes has been aided materially by this paper. This is particularly true from an academic point of view as the sand models were necessarily confined to a definite boundary at the base. The models were not influenced by part or full saturation and therefore could not be expected to incorporate the complexities which seem to be typical of slides in prototypes such as dams, canal banks, and mountain slopes. An ingenious device was used to simulate "cohesion" in the materials used in the soil tests.

Sand Model Tests.—The application of atmospheric pressure on the exposed surfaces of the sand models does not truly represent cohesive action. It represents the compressive effect of these outer forces. Hence, it lends support to these surfaces and causes consolidation of the soil and, thereby, changes the stress distribution that would be established in the presence of true cohesive forces in the prism. The latter would be mutually balanced within the mass without the aid of outer forces.

To avoid confusion, the term "outside pressure" will hereafter be used in referring to the simulated cohesion. It must also be kept in mind that the lateral pressures in the prism are affected, to some extent, by the sloping top which is established in the course of tilting the model, and that the shear distribution along the inclined base is likewise affected. In the course of this discussion the effect of outside pressure and of tilting prisms will be further studied and compared with the conditions that would establish themselves without tilting.

It would have been helpful if the author had shown a screen analysis and had furnished information on the density and moisture content of the material in question. It is hoped that this will be done in the closing discussion to complete the paper.

A most interesting and valuable feature was brought out in the course of the tests on the sand models—the relation of depth to the limit of disturbance

NOTE.—This paper by Ek-Khoo Tan was published in January, 1947, *Proceedings*.

[15] Asst. Chf. Engr., Los Angeles Flood Control Dist., Los Angeles, Calif.

[15a] Received March 24, 1947.

and to the surface of rupture. The insight into the mechanics of sliding failure and its division between a plastic and an elastic zone is thereby enhanced.

Prototype slides, however, are usually related to soil moisture, and to the pore water stress that results from it. This is particularly true of slopes that are exposed to water subject to fluctuation. With few exceptions, landslides observed in nature have likewise been closely linked with the moisture content in the soil and the stresses caused by this moisture, either static or dynamic. The tests described in the paper naturally do not shed any light on these phenomena; they are limited entirely to gravitational influences on the soil itself, and to the pressure acting on the outer surface.

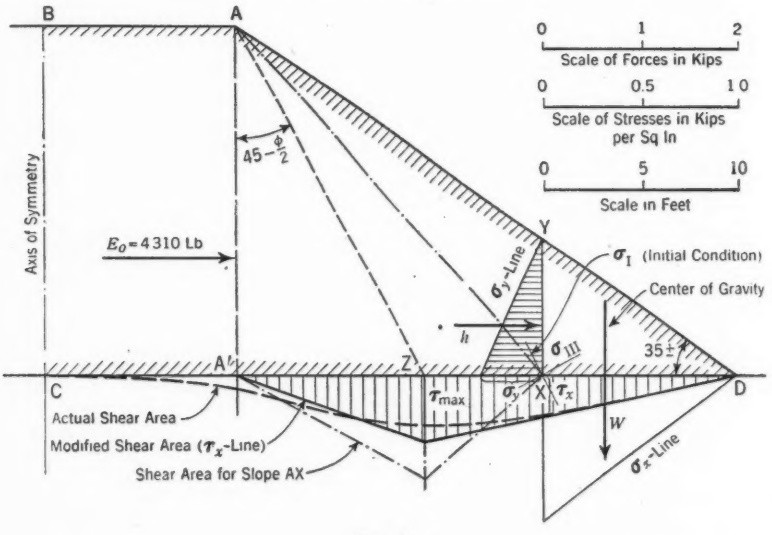


FIG. 14

The problem of analyzing the initial stress distribution in a soil prism, such as that used in the tests, has never been uniquely solved. This solution may be approached in the following manner:

Let line CB in Fig. 8 be the axis of symmetry of an embankment, the dimensions of which are equal in feet to the values specified in inches by the author. Then, distance CD is equal to one half of its base. The shear distribution along the base may be approximated[16] in distributing the later pressure E_o as shown in Fig. 14. Actually, the shear is distributed as indicated by the dashed line. The hatched area then represents the total lateral pressure per unit width of prism acting on a vertical plane through point A (see Fig. 14). Its value is ascertained by the Coulomb formula for cohesionless granular soil in:

$$E_o = \frac{\gamma h^2}{2} \tan^2 \left(45 - \frac{\phi}{2} \right) \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

[16] "Soil Mechanics," by D. P. Krynine, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1941, p. 272.

in which γ, the unit weight of soil, equals 90 lb per cu ft (assumed); ϕ, the angle of friction, equals 33° (according to Mr. Tan); and $h = \overline{AA'}$.

It is significant to note that maximum, modified shear occurs along line AZ, which means that it also occurs near the top of the slope. This was confirmed by the test. The rapid increase in the shear intensity with the steepening of the slope is also significant (see dashed and dotted line in Fig. 14). Tilting the prism causes an increase in the shear intensity which is even more pronounced in that E_o increases with the tilt. Thus, tilting the base mobilizes forces which are different from those created by the steepening of the slope

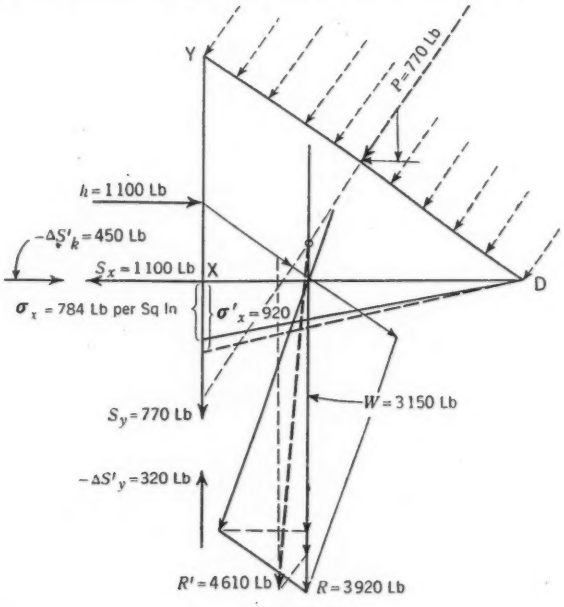


FIG. 15

alone. Next, consider a small triangular prism near the toe of the slope which, in the initial state, is held in equilibrium by the stresses σ_x, σ_y, τ_x, τ_y, and the body force W.

The triangle formed with σ_y as the base represents the horizontal force H which is equal and opposite to the shear S_x between points X and D in Fig. 14. The vertical (downward) shearing stress τ_y at point X is equal to τ_x. Hence, the vertical shear S_y may be determined. The weight of the prism is W. The prism is drawn to a large scale in Fig. 15 and the stresses and forces applied. The bearing pressure σ_x at point X must be determined before the principal stresses can be ascertained. This is done by combining H and S_y, and W and S_x, into resultants and by combining the two. The final resultant R falls closely in line with W and permits the σ_x-stress at point X to be determined. This stress is not equal to the pressure caused by the height XY of the prism. Fig. 16 shows the determination of the principal stresses and their directions by Mohr's circle. These directions are transferred to point X in Fig. 14. As

a boundary condition for the initial state all stresses must be zero along line AD.

Now, let the outside pressure P be applied uniformly so as to correspond to 1 ft of water column, equal to 1 in. in the model test as shown in Fig. 15 by dashed lines. Clearly, the stress condition along the surface has changed; the normal stress is no longer zero, nor is the longitudinal stress necessarily along the outside fiber. Combining P with R gives a new resultant R' which reduces τ_x and τ_y and increases σ_x. The corresponding principal stresses and their directions are shown (by the dashed line) in Fig. 16.

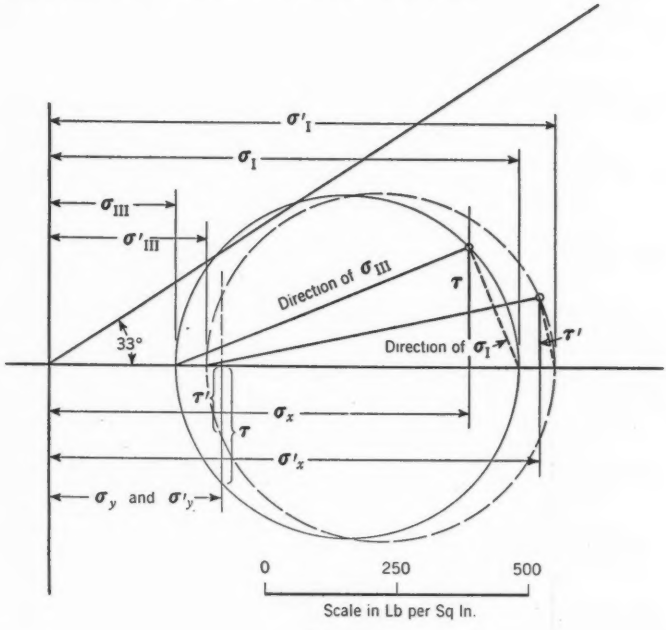


FIG. 16

If the outside pressure P were increased indefinitely, the soil mass would eventually become a solid, and the section could be treated as a monolith. The stress condition would then again undergo a complete change. Hence, the application of outside pressure creates stress conditions different from those resulting from cohesion. It primarily creates a pre-stressed condition, as discussed by the writer in connection with the San Gabriel Dam No. 1 of the Los Angeles County (Calif.) Flood Control District.[17]

A better simulation of cohesion than that achieved by using the rubber membrane over the outer surface and the part vacuum in the voids of the soil could have been accomplished with mass forces created by the rotation of the model at various speeds around a center. This would have had the advantage of eliminating outer forces, since the mass (centrifugal) forces would naturally apply to all the particles of the sand mass simultaneously.

[17] "Design and Construction of San Gabriel Dam No. 1," by Paul Baumann, *Transactions*, ASCE, Vol. 107, 1942, pp. 1648–1651.

Another more direct method would have been the use of soil cement of various mixes. Actual cohesion and internal friction could then be determined by tests.

Swedish Circular Arc Analysis.—It appears as if the discrepancy in the safety factor at failure in the Swedish circular arc method of analysis may be due to two principal factors: (1) The angle of internal friction evidently was considered to be equal to the minimum angle of repose of 33°, whereas, in effect, it probably was greater; and (2) the analysis apparently was made on the sand models that had been subjected to outside pressure on the rubber membrane. These forces should be superposed on the gravitational and lateral forces and the stresses along the rupture surface determined in similar manner, as outlined by the writer in connection with the soil prism (see Fig. 15). Apparently, the outside pressure was simply assumed to increase normal components along the rupture surface without simultaneously decreasing the tangential components. Thus, although factor (1) would cause an increase in the numerator, factor (2) would have the opposite effect on the denominator, and the two combined might well result in a factor of safety approaching unity.

Photoelastic Tests.—It is somewhat difficult to agree as to the "remarkable resemblances" between the results of the photoelastic tests and those of the sand tests, particularly in view of the fact that the photoeleastic tests do not show the formation of the surface of rupture. The shear distribution in the gelatin models is quite at variance with the one in the sand models except for the fact that both show a wave pattern.

Theoretical Plasticity Analysis.—Like most researchers in the field of soil mechanics, the author had to make considerable concessions, particularly in the way of approximations, to avoid terms of hopeless complexities for analytical solution. This circumstance, no doubt had much to do with the fact that Karl Terzaghi, M. ASCE., has defined soil mechanics as a descriptive science. The mathematical analysis is nevertheless remarkable, although the results, as shown in Fig. 13, bear little resemblance to either the Swedish circular arc or the photoelastic analyses.

Conclusion.—There is no question in the writer's mind but that the author's work will lend itself advantageously for further advance in the knowledge of the idiosyncracies of soil slopes, either natural or artificial.

GREGORY P. TSCHEBOTARIOFF,[18] M. ASCE.[18a]—A noteworthy feature of this paper is the attempt to apply the mathematical analysis of stress conditions within a soil mass by means of the theory of plasticity. The results of the several methods of investigation, as presented in the paper, do not appear to lead to exactly the same conclusions. However, such differences as have been registered are of value since they serve to reveal the limitations of some methods of investigation.

Fig. 3 presents, in clear graphical form, the fact that a surface of failure can develop only at some distance from a rigid boundary after the shearing strains have reached a certain value. This is a very significant finding. These shear-

[18] Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

[18a] Received March 27, 1947.

ing strains develop within the soil mass beneath the surface of failure in a soil zone which is frequently assumed as not affecting the location of the surface of failure.

Fig. 5 shows that the maximum strains occur close to the toe of the slope as determined from the displacements of lampblack lines caused by motions of the sand grains.[18b] In this respect, these curves appear to be in agreement with the general trend of stress distribution represented in Fig. 9, which summarizes the results of the photoelastic studies. There again the greatest stress concentration is shown to occur at the toe of the slope. It seems to the writer that this is a direct consequence of the presence of the rigid boundary along the base of the model slopes, and of the fact that the inclination of this boundary varied as a function of the slope i and increased with it. Judging from the isochromatic patterns shown in Figs. 9(*a*) and 9(*b*), the lubrication of the base did not noticeably reduce the stress concentration at the toe of the slope, and even appears to have increased it somewhat. In both cases (as stated by the author in Part II, section 2), the stress concentration at the toe of the slope must have been strongly influenced by the separation of the gelatin from the rear vertical face BC of the flume (see Fig. 8) which prevented the development of tension at that boundary.

When dealing with natural slopes in the field, one usually encounters conditions in which the cohesive soil mass extends well beyond the toe of the slope both in the vertical and in the horizontal directions. It is doubtful that stress concentrations of an intensity similar to the one shown in Fig. 9 could occur then, irrespective of whether tension cracks had, or had not, developed near the top of the slope. Accordingly, in the writer's opinion, one of the most interesting and significant contributions of the paper lies in the fact that the considerable importance of boundary conditions is made apparent, as well as the influence of these boundary conditions on the redistribution of stresses within the soil mass, and on the location of the surface of failure.

[18b] Correction: In January, 1947, *Proceedings*, p. 25, lines 10 and 17, change "top of the slope" to "middle of the sliding curve."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FORECASTING PRODUCTIVITY OF IRRIGABLE LANDS

Discussion

BY ARTHUR F. JOHNSON, H. W. VAN LOO, AND WILL H. NOBLE

ARTHUR F. JOHNSON,[6] ASSOC. M. ASCE.[6a]—An ingenious method of estimating average crop yields from the relation of such yields to available heat in day degrees Fahrenheit using the method for computing available heat presented by Robert L. Lowry, Jr., M. ASCE, and the writer[2] is suggested by the author. The method will be useful for reconnaissance studies where data on crop yields under irrigation are not readily available. It is not sufficiently reliable to serve as a close criterion of project feasibility. As the author wisely states, the method is more limited in estimating productivity than in estimating consumptive use of water due to the greater number of factors contributing to the variability of the results.

The author has confined his treatment to individual crops but has related yields to the average available heat rather than to individual effective heat values for each crop as suggested by C. R. Hedke,[7] M. ASCE. Somewhat more reliable results might be expected by developing separate curves using appropriate temperature bases for each crop.

The yield curves for both alfalfa and potatoes (Fig. 1) are surprisingly well supported by the plotted points representing the areas studied. The marked drop in yield beyond optimum heat conditions is characteristic of most crops. Alfalfa grows continuously where temperatures do not drop below freezing but in the Imperial Valley (California) and in other areas of extremely high sustained summer temperatures its growth is inhibited during the hottest part of the season and it becomes almost dormant. The growing season for potatoes, on the other hand, is comparatively short and it is actually possible to produce two crops in a single year although this would not be an acceptable

NOTE.—This paper by the late W. C. Muldrow was published in February, 1947, *Proceedings*.

[6] Acting Chf., Div. of Eng. and Surveys, Branch of Project Planning, Bureau of Reclamation, Washington, D. C.

[6a] Received March 17, 1947.

[2] "Consumptive Use of Water for Agriculture," by Robert L. Lowry, Jr., and Arthur F. Johnson, *Transactions*, ASCE, Vol. 107, 1942, p. 1243.

[7] "Consumptive Use of Water by Crops," by C. R. Hedke, New Mexico Engr.'s Office, July, 1924 (unpublished).

practice because of the drain on soil fertility and risk of fostering potato diseases. It is presumed that the potato curve in Fig. 1 applies to a single planting so that materially higher total yields might be expected if repeated crops in the same year were included where climatic conditions permit. Nevertheless, as the limit of tolerance to heat is approached, yields may be expected to fall off to a negligible amount.

In Fig. 1, the curve "Consumptive Use per Ton of Alfalfa" was evidently derived by dividing values on the "Consumptive Use Curve—Valley Average" by corresponding points on the alfalfa yield curve. This procedure is not proper because the curve of average valley consumptive use represents a composite of consumptive use requirements for crop distributions typical of each heat condition. The percentage of alfalfa is not the same throughout the range of the consumptive use curve but rises from zero in frosty mountain meadows to a maximum and then drops with increasing heat. The percentage of alfalfa is high for the Snake River Valley (Idaho) but is relatively low for the Mesilla Valley (New Mexico) where cotton is "king." Therefore, the derived curve is somewhat lower than would be the case with true values of consumptive use of alfalfa, and the error increases in some degree with increasing heat as alfalfa is a relatively higher consumer of water than the average crop distribution which the Lowry-Johnson curve represents.

H. W. VAN LOO,[8] JUN. ASCE.[8a]—The heat-unit method of estimating the consumptive use of water for agriculture extended to provide a method of estimating the productivity of lands on a proposed project may provide a very satisfactory system of forecasting the returns to any proposed development. However, the relatively low yield of alfalfa in the Imperial Valley (California), about which the late Mr. Muldrow expressed surprise, may not be entirely justifiable.

TABLE 1.—ANNUAL YIELD OF ALFALFA, IN TONS (TOTAL) AND TONS PER ACRE (UNIT), FOR COACHELLA VALLEY, CALIFORNIA

Year	Acreage	Total yield	Unit yield
1939	1,276	5,104	4.0
1940	1,400	9,100	6.5
1941	1,600	12,000	7.5
1942	1,100	5,500	5.0
1943	1,310	9,170	7.0
1944	1,243	8,079	6.5
1945	1,541	7,705	5.0
1946	1,741	12,187	7.0
Average	...	...	(6.1)

Although the writer does not propose to question the total yield of alfalfa shown for Imperial Valley, there may have been some misinterpretation of conditions. In a region such as Imperial Valley where alfalfa is used on considerable acreage for rejuvenation of the soil, the total yield cannot be divided by the total acreage under cultivation, since the total acreage is not always producing alfalfa during an entire year. A second factor that may have affected the results is that a considerable part of the total acreage is used for pasturing various kinds of livestock. For instance, range cattle may be purchased in Arizona, shipped to Imperial Valley, and fattened on alfalfa. More than likely the acreage used for pasturing has been included in the total acreage since some cutting of alfalfa is usually made, but no correction has been made for the decreased cuttings of alfalfa.

[8] Civ. Engr., Coachella, Calif.
[8a] Received March 17, 1947.

As supporting data for the contention that some error has been made, Table 1 shows the yield of alfalfa for near-by Coachella Valley which is about sixty miles north of Imperial Valley and has nearly identical weather conditions.

The average yield of 6.1 tons of alfalfa per acre for Coachella Valley is about twice as high as the 2.99 tons shown for Imperial Valley. Possibly, the light sandy soils which predominate in Coachella Valley plus the generally well-drained farm lands would account for some of the difference. Whatever the reason for the discrepancy, it is suggested that further examination be made of the yield of crops grown in the warmer sections of the United States.

WILL H. NOBLE,[9] Assoc. M. ASCE.[9a]—The law of a heat-productivity relation is set forth admirably in this paper. Perhaps the popular concept that water makes the crop should give way to a concept of heat as the productive agent, with commensurate water an adjunct. The author's demonstrated heat-productivity relation is one of a succession of relations applying to the actual problem of economic feasibility, namely: (1) Water requirement varies with heat, (2) crop production varies with heat, (3) crop value varies with production, hence with heat, and (4) economic feasibility is a function of crop value, hence a function of heat. If procedure toward item (4) is necessarily by way of item (2), the methods of finding the (2) relation should be explored for possible improvement and standardization.

For instance, the heat-productivity relations studied and graphed for the paper appear to be based on the 1940 Census; that is, on 1939 crops, and (presumably)1939 seasonal heat. Since economic feasibility rests on a balance of average annual costs versus average net returns, ratios of average yield to average seasonal heat are required. A compilation of average heat-yield relations for the commonly irrigated crops, with probable deviations from such averages, and a recording of the most favorable heat range for each crop, would be a valuable addition to engineering statistics. It could be based on the United States Weather Bureau value for normal mean monthly temperatures above 32° F, and the annual reports of crop yields and values on United States Bureau of Reclamation projects, so far as they have been published. These crop reports cover existing projects in a block of fifteen states extending eastward from the Pacific coast to include North Dakota, South Dakota, Nebraska, Colorado, New Mexico, and Texas on the east. Deductions from so great a range of data should be of general application in any part of the United States where irrigation may be practiced or proposed. This is true whether the proposal is purely for irrigation or is a multiple-purpose project with irrigation one of a number of interests.

The widely accepted Lowry-Johnson system of heat units, in day degrees of maximum temperatures above 32° F, has quite naturally been used by the author. This system was evolved for solving the heat versus consumptive-use-of-water relation, closely akin to evaporation. For this purpose, use of the maximum daily temperature was found to give a closer correlation than was found with average daily temperature. Unless a similarly superior correlation is

[9] State Center, Iowa.

[9a] Received April 1, 1947.

found for the heat versus productivity relation by use of the maximum temperatures, it would appear a simpler and more practical procedure to refer directly to the normal mean monthly temperatures of the Weather Bureau's annual summaries, and to arrive directly at the normal seasonal heat in degree months. In this case, the productivity season would naturally terminate with the average date of first killing fall frost.

Since heat determines yield, and yield determines crop value per acre, should there not be a definite heat versus value-per-acre relation? Increased heat means larger crops, or a greater number of crops per season, or higher value crops, all of which mean more dollars per acre, for more heat per acre. Project income is a composite income from a diversity of crops whose seasonal heat ranges include the seasonal heat of the particular project. Must the engineer determine probable yield and price for each crop, then assume some pattern of crop diversity, and so estimate the composite crop value? Or does nature itself determine the diversity largely through the available heat? Can a heat versus crop-value graph be drawn that will give the desired preliminary estimate of farm income? The writer's study of Bureau of Reclamation crop statistics for the decade from 1921 to 1930, inclusive (the most stable decade of reclamation history as to commodity prices and construction costs), indicates that such a relation may be definitely charted, with correlations that favorably compare with those for the heat-productivity relation. If so, such a chart or graph should be a logical sequence to the main argument of the paper. For its application, the general price structure would need to be projected forward to the years of construction and operation of the project under consideration; but this must be done whether prices are to be applied item by item, or as a composite per-acre figure.

This discussion is intended to imply acceptance and appreciation of the paper, and a desire to see the idea extended and put into most usable forms. Whatever the form in which it is expressed, the author's well-worded caution against undue reliance on the average ratios found will be applicable. The individual irrigator may not safely assume that his own farm will live up (or down) to the project average income per acre; nor can the irrigation engineer assume that his own project will be an average one. That point he must determine by more particular investigation.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRENGTH OF BEAMS AS DETERMINED BY LATERAL BUCKLING

Discussion

BY H. G. BRAMELD

H. G. BRAMELD,[39] ESQ.[39a]—An excellent case is made for approximate formulas for critical and working buckling stresses, in so far as rolled steel joists are concerned. Unfortunately, agreement with the fundamental equation (Eq. 4) is not good in the case of plate girders with thin flanges. Startling results were obtained when the equations were applied to the girders of a bridge in which the writer is interested.

The girders vary in depth from 86 in. to 170 in. and are braced laterally at intervals of 22 ft. Their properties are as follows:

Case I.—Depth: 86 in. Make-up: Two flange plates, 18 in. by $\frac{3}{8}$ in.; four flange angles, 8 in. by 8 in. by $\frac{5}{8}$ in.; and one web plate, 84 in. by $\frac{3}{8}$ in. For this section, $I_y = 818$ in.^{4}; $I_x = 108{,}000$ in.^{4}; and the torsion constant = 26.6. The equivalent thickness of the compression flange is found to be $\frac{6\,I_y}{b^3} = 0.84$ in. From Eq. 7, $f = \frac{20{,}000{,}000}{\frac{l\,d}{b\,t}} = \frac{20{,}000{,}000}{264 \times 86} = 13{,}400$ lb per sq in. However, by extrapolation of k in Table 1, with $a = 0.806\,d\sqrt{\frac{I_y}{k}} = 380$ in., and $k = 2.9$, Eq. 4 yields $f = \frac{818 \times 86^2}{108{,}000 \times 380^2} \times 29{,}000{,}000 \times \frac{380^2}{264^2} \times 2.9 = 67{,}000$ lb per sq in., or just five times the approximate result.

Case II.—Depth: 170 in. Make-up: One web plate, 168 in. by $\frac{3}{8}$ in., with flanges the same as in case I. By Eq. 7, $f = 6{,}750$ lb per sq in. and, by Eq. 4, $f = 48{,}000$ lb per sq in.

NOTE.—This paper by Karl de Vries was published in September, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1946, by George Winter, and David B. Hall; February, 1947, by Theodore R. Higgins; March, 1947, by Neil Van Eenam; and April, 1947, by H. N. Hill, and H. D. Hussey.

[39] Designing Engr., Bridge Board, Brisbane, Queensland, Australia.

[39a] Received March 3, 1947.

Case III.—Depth: 86 in. Make-up: Six flange plates, 18 in. by $\frac{3}{8}$ in.; four flange angles, 8 in. by 8 in. by $\frac{5}{8}$ in.; and one web plate, 84 in. by $\frac{3}{8}$ in. By Eq. 7, $f = 21{,}000$ lb per sq in. and, by Eq. 4, $f = 90{,}000$ lb per sq in.

Case IV.—Depth: 170 in. Make-up: One web plate, 168 in. by $\frac{3}{8}$ in., with flanges the same as in case III. By Eq. 7, $f = 10{,}700$ lb per sq in. and, by Eq. 4, $f = 64{,}000$ lb per sq in.

These girder flanges when treated as struts were found to have a safe stress between 16,000 lb per sq in. and 17,000 lb per sq in., consistent with an intrinsic eccentricity of 0.45 in. and an extreme fiber stress of 20,000 lb per sq in. It is apparent, therefore, that the equation for safe working stresses cannot reasonably be applied to deep plate girders and that this fact should be noted in codes using the new formula.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RELIEF WELLS FOR DAMS AND LEVEES

Discussion

BY REGINALD A. BARRON, PRESTON T. BENNETT, CHARLES I. MANSUR, AND S. J. JOHNSON

REGINALD A. BARRON,[30] JUN. ASCE.[30a]—Practical and theoretical requirements for the design of relief wells have been presented in this paper. Of necessity both these requirements must be considered in a design. The general case presented by the authors consists of a semi-infinite pervious layer of uniform thickness covered by an absolutely impervious blanket. In nature the blanket is always slightly pervious and in some cases, such as at Sardis Dam, Mississippi, the downstream blanket is of finite length.

The writer has obtained a mathematical solution for this case that meets the boundary conditions as shown in Fig. 22(*b*). The hydraulic head h_{xy} measured from a datum plane is

$$h_{xy} = h_p - (h_p - h_T)\frac{y}{L} - \bar{h}_{xy} \quad \ldots\ldots (35)$$

in which

$$\bar{h}_{xy} = q \sum_{n=-\infty}^{n=+\infty} \log_e \left[\frac{\cosh \frac{\pi}{L}(n\,a - x) - \cos \frac{\pi}{L}(y + s)}{\cosh \frac{\pi}{L}(n\,a - x) - \cos \frac{\pi}{L}(y + s)} \right] \quad \ldots\ldots (36)$$

in which n takes on all integral values between $-\infty$ and $+\infty$ including zero; and

$$q = \frac{\bar{h}_w}{\log_e \left[\frac{1 - \cos \frac{2\pi s}{L}}{1 - \cos \frac{\pi r_w}{L}} \right] + 2 \sum_{n=1,2,3,\ldots}^{n=\infty} \log_e \left[\frac{\cosh \frac{\pi n a}{L} - \cos \frac{2\pi s}{L}}{\cosh \frac{\pi n a}{L} - 1} \right]} \quad \ldots (37)$$

NOTE.—This paper by T. A. Middlebrooks and William H. Jervis was published in June, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1946, by Henry C. Barksdale, Willard J. Turnbull, and Glennon Gilboy; December, 1946, by W. A. Wall and C. A. Stone; January, 1947, by John R. Charles, Horace A. Johnson, P. C. Rutledge, H. H. Roberts and Carter V. Johnson, Frank E. Fahlquist, and Harry R. Cedergren; and February, 1947, by John S. McNown.

[30] Engr., Embankment and Foundation Branch, U. S .Waterways Experiment Station, Corps of Engrs., Vicksburg, Miss.

[30a] Received March 24, 1947.

Approximating $\frac{1}{1 - \cos \frac{\pi r_w}{L}}$ as $\left(\frac{2}{\pi^2} \frac{L^2}{r^2_w}\right)$, Eq. 37 may be rewritten as:

$$q = \frac{\bar{h}_w}{f\left(\frac{L}{r_w}\right) + f\left(\frac{s}{L}\right) + F\left(\frac{s}{L}, \frac{\pi a}{L}\right)} \quad \ldots\ldots\ldots\ldots (38)$$

in which

$$f\left(\frac{L}{r_w}\right) = \log_e \left(\frac{2}{\pi^2} \frac{L^2}{r^2_w}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

$$f\left(\frac{s}{L}\right) = \log_e \left(1 - \cos \frac{2\pi s}{L}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

and

$$F\left(\frac{s}{L}, \frac{\pi a}{L}\right) = 2 \sum_{n=1,2,3,\ldots}^{n=\infty} \log_e \left\{ \frac{\cosh \frac{\pi n a}{L} - \cos \frac{2\pi s}{L}}{\cosh \frac{\pi n a}{L} - 1} \right\} \quad \ldots\ldots (41)$$

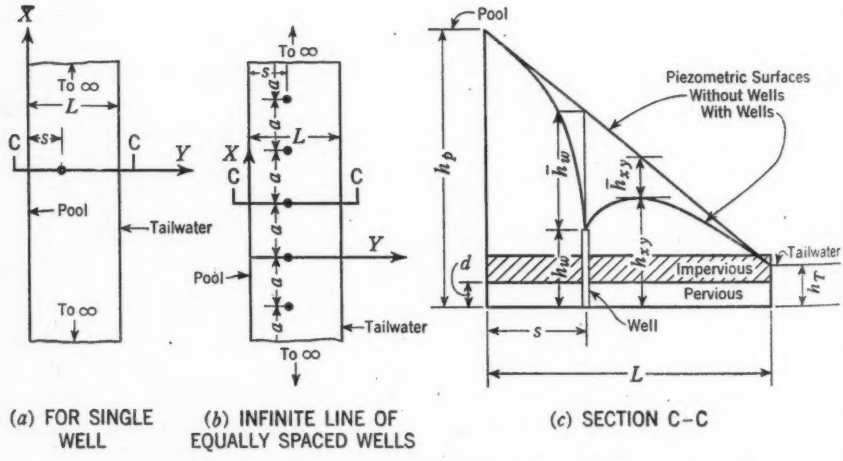


FIG. 22.—SINGLE WELL AND INFINITE LINE OF EQUALLY SPACED WELLS FOR CASE OF BLANKET WITH FINITE WIDTH

The individual well discharge is

$$Q = 4\pi k q d \quad \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

and the head acting on the well is

$$\bar{h}_w = h_p - (h_p - h_T) \frac{s}{L} - h_w \quad \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

The head above the well discharge midway between wells is $\bar{h}_w - \bar{h}_m$. Of

these two values, h_m is expressed as:

$$h_m = q \sum_{n=-\infty}^{n=+\infty} \log_e \left[\frac{\cosh \frac{\pi a}{L}(n - \frac{1}{2}) - \cos \frac{2\pi s}{L}}{\cosh \frac{\pi a}{L}(n - \frac{1}{2}) - 1} \right] \quad \ldots\ldots\ldots (44)$$

in which n takes on all integral values between $-\infty$ and $+\infty$ including zero. The head $h_w - h_m$ may then be written as

$$h_w - h_m = \left[1 - \frac{f\left(\frac{s}{L}, \frac{\pi a}{L}\right)}{f\left(\frac{L}{r_w}\right) + f\left(\frac{s}{L}\right) + F\left(\frac{s}{L}, \frac{\pi a}{L}\right)} \right] h_w \ldots\ldots (45)$$

In Eq. 45, $f\left(\frac{s}{L}, \frac{\pi a}{L}\right)$ is the quantity within the braces of Eq. 44. Curves for

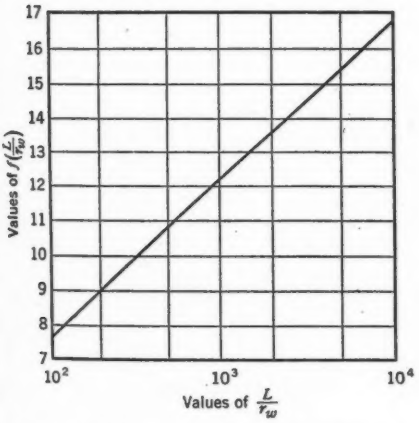


FIG. 23.—CURVE FOR THE SOLUTION OF EQ. 39

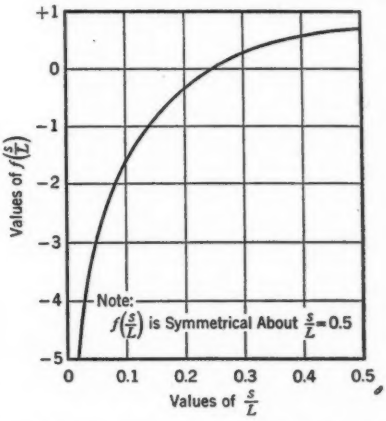


FIG. 24.—CURVE FOR THE SOLUTION OF EQ. 40

Eqs. 39, 40, and 41 and the quantity $f\left(\frac{s}{L}, \frac{\pi a}{L}\right)$ in Eq. 45 are given in Figs. 23, 24, and 25. For $\frac{a}{L} = \infty$, the values within the summation signs of Eqs. 36, 37, 41, and 44 are zero except for $n = 0$ and the case reduces to that of a single well as shown in Fig. 22(*a*). The value h_{xy} is then

$$h_{xy} = q \log_e \left[\frac{\cosh \frac{\pi x}{L} - \cos \frac{\pi}{L}(y + s)}{\cosh \frac{\pi x}{L} - \cos \frac{\pi}{L}(y - s)} \right] \quad \ldots\ldots\ldots\ldots (46)$$

and q may be determined by use of Eqs. 39 and 40 (Eq. 41 being equal to zero).

The average gradients to the well for the case of equal pool and tailwater may be approximated as follows by equating the well flow to that flowing to a continuous slot at s:

$$Q = 4\,\pi\,k\,q\,d = k\,h'\,a\left(\frac{1}{s} + \frac{1}{L-s}\right)d \quad \ldots\ldots\ldots\ldots (47)$$

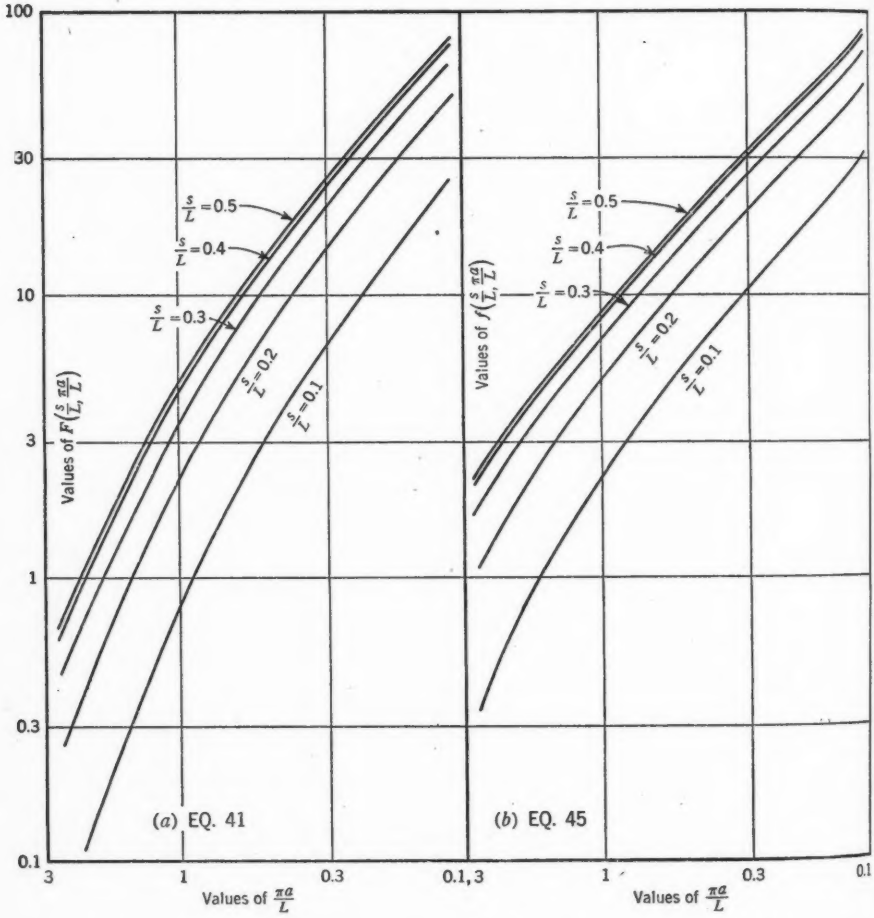


FIG. 25.—CURVES FOR THE DETERMINATION OF $F\left(\frac{s}{L}, \frac{\pi a}{L}\right)$ IN EQ. 41 AND $f\left(\frac{s}{L}, \frac{\pi a}{L}\right)$ IN EQ. 45

The effective head for the slot is

$$h' = 4\,\pi\,q\,\frac{s\,(L-s)}{a\,L} \quad \ldots\ldots\ldots\ldots (48)$$

The gradient out of the pool is then

$$g_p = \frac{h'}{s} = 4\,\pi\,q\,\frac{L-s}{a\,L} \quad \ldots\ldots\ldots\ldots (49)$$

whereas the gradient from the tailwater is

$$g_t = \frac{h'}{L - s} = 4\,\pi\,q\,\frac{s}{a\,L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

For the case in which pool and tailwater are not at the same elevation:

$$g_p = \frac{h_p - h_T}{L} + 4\,\pi\,q\,\frac{L - s}{a\,L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

and

$$g_t = \frac{h_p - h_T}{L} - 4\,\pi\,q\,\frac{s}{a\,L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

It is readily apparent that the well discharge and the uplift midway between the wells for the foregoing case is not a function of the head difference $(h_p - h_w)$. Variation of the tailwater elevation for constant pool elevation has an important effect on well discharge observed at the Sardis Dam relief wells. An example is shown in Fig. 26. The physical data are given in

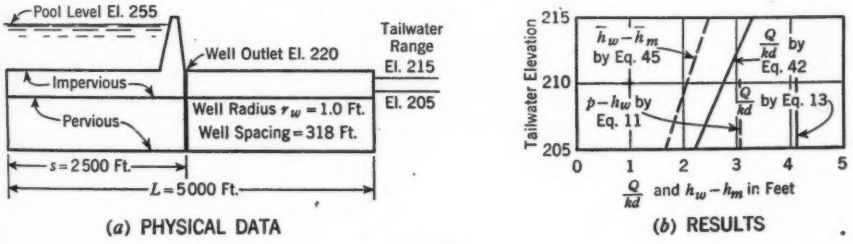


FIG. 26

Fig. 26(*a*) and the result in Fig. 26(*b*). As should be expected, the relief measure furnished by the tailwater outlet reduces both the well discharge and the uplift pressure midway between wells as compared with those obtained by using Eq. 11 and 13. As the value of $\frac{s}{L}$ becomes smaller these differences become smaller. It is thus apparent that, for a project having excessive foundation seepage pressures and also a downstream outlet, the effectiveness of relief wells should be determined as outlined in this discussion rather than by the methods given by the authors.

PRESTON T. BENNETT,[31] M. ASCE.[31a]—This study of pressure relief is admirably balanced between a general discussion of basic principles involved, and the specific analytical methods that may be used in the application of these principles to the design of an installation.

With regard to the uncertainties concerning the boundaries of flow, permeability coefficients, and the effects of anisotropic permeability, it is believed that extensive field investigations of these factors may or may not be justifiable, depending on the type of installation. If the wells are to be installed at the

[31] Senior Engr., Corps of Engrs., Omaha Dist., Omaha, Nebr.

[31a] Received March 24, 1947.

toe of a dam, flexibility of the system can be insured by placing them in an accessible location, and by omitting any elaborate collection system with a fixed discharge capacity. As the authors have stated in the paper, any deficiency in the original installation can be remedied by the addition of more wells when, and if, actual operation shows it to be necessary.

If, on the other hand, a deep subdrainage system is being considered for prevention of boils behind a levee, the inherent flexibility of vertical well drainage cannot always be fully utilized. Deep drainage would seldom be considered in connection with a levee unless the levee protected highly developed urban or industrial areas, where discharge of the well flow on the land-side surface would not be permissible. In such a situation, design of the pressure-relief system involves the determination of the required capacity of the collection and disposal system, and the cost of these facilities may justify fairly expensive field studies.

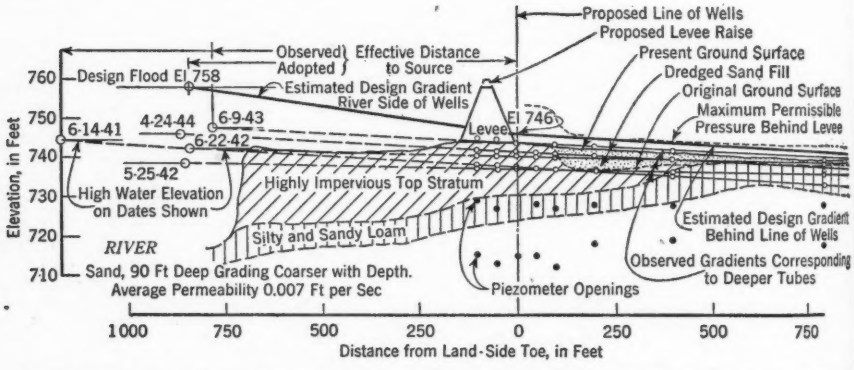


FIG. 27.—SECTION THROUGH FAIRFAX LEVEE AT KANSAS CITY, KANS., AT PIEZOMETER INSTALLATION

Two types of field investigations are suggested. First, to determine approximately the boundaries of flow, liberal use may be made of piezometers installed in the pervious substratum. Interpretation of piezometer observations is complicated because ordinarily they reflect nonsteady states of flow resulting from fluctuations of river stage. However, if the period of observations includes several stages of sufficient height and duration to fill or nearly fill underground storage, the field observations can be of great direct value in design. Fig. 27 indicates how the effective distance to the source of flow may be determined by piezometer observations. It shows data obtained by an installation at Kansas City, Kans., behind a levee constructed by the Corps of Engineers. The general relationship between the foundation pressure gradient, the rate of rise of the river, and the duration of the high stage may also be studied by use of observed data of the type shown in Fig. 27.

For a fairly accurate estimate of foundation permeabilities, deep well pumping tests are considered indispensable. They furnish directly the information needed in design, which is the quantity of water that will move through the entire foundation depth under a given gradient. Existing water-supply wells may be available for this purpose, or it may be necessary to install wells as a

part of the test. In the latter case, the test can be used to evaluate the performance of the type of well screen proposed for the system being designed.

It is also suggested that well pumping tests afford the only means of evaluating the factor of partial penetration in a stratified formation. The authors have shown, by model tests with an isotropic medium, that the percentage of penetration is more important than well size or spacing. In an actual foundation, penetration undoubtedly is of even greater importance, but it cannot be measured solely in terms of screen length versus sand depth, without reference to the comparative water-bearing capacities of the strata penetrated. A series of pumping tests at one location, in which the penetration of the screen is varied by successively filling and plugging off lower sections, will give directly a correction factor for penetration which can be applied to full penetration formulas. The applicability of this factor to other locations would of course be questionable, and should be checked by borings throughout the proposed system for comparison of the degree of stratification at various points.

Considering the difficulty in determining the flow boundaries and permeabilities involved in an actual foundation, and consequent inability to use accurate constants in design formulas, it may seem inappropriate to suggest modifications to the analyses presented. Certainly no greater numerical accuracy will be obtained. However, further discussion of theoretical design methods is not inconsistent with the authors' presentation of theory as an aid to judgment in arriving at a reasonable solution for an actual problem.

The writer has had frequent occasion during the past five years to apply the authors' methods to studies of proposed well installations for levees at several locations on the Missouri River, and to compare theoretical with actual well performance as exemplified by the installation at Fort Peck Dam in Montana. As a result of these studies, the writer proposed in 1944[13] a modification of the well-spacing formulas developed by the U. S. Waterways Experiment Station (Vicksburg, Miss.). The modification is, in most cases, of minor importance in its effect on numerical results of design computation; but it is fundamental in so far as it concerns the relation between the wells (or any other deep drainage system) and the over-all flow pattern in the foundation. Since the deviation of the modified formula has been published elsewhere,[13] this discussion will be concerned principally with its more general aspects.

The only basic difference between the writer's analysis and that of Messrs. Middlebrooks and Jervis is that a regional flow landward from the well system is introduced. Perhaps the omission of this flow from the early studies was due to the use of models as shown in Fig. 5, which had an impervious bulkhead a short distance landward from the wells. In an actual installation, the pressure head in the region of the wells that causes them to flow ordinarily also will cause a landward flow away from the wells. This flow may or may not be important in any given case, but should be included in any general theoretical solution.

To simplify the discussion, a very simple model with definite dimensions will be substituted for the actual problem. Fig. 28 represents such a model,

[13] "Comments on the Design of Relief Wells," by P. T. Bennett, Rept. on Conference on Control of Underseepage, U. S. Waterways Experiment Station, Vicksburg, Miss., April 1, 1945.

and shows the notation used in this discussion. It represents a pervious medium of depth Z and length L_0, covered by an impervious layer which has a few openings filled with pervious material. These openings in the model represent places in the prototype where inflow or outflow from the pervious sand is relatively easy, as, for example, in borrow pits which partly expose the sand. They may also represent seepage through a surface blanket of low permeability. In this case the foundation pressure profile would become a

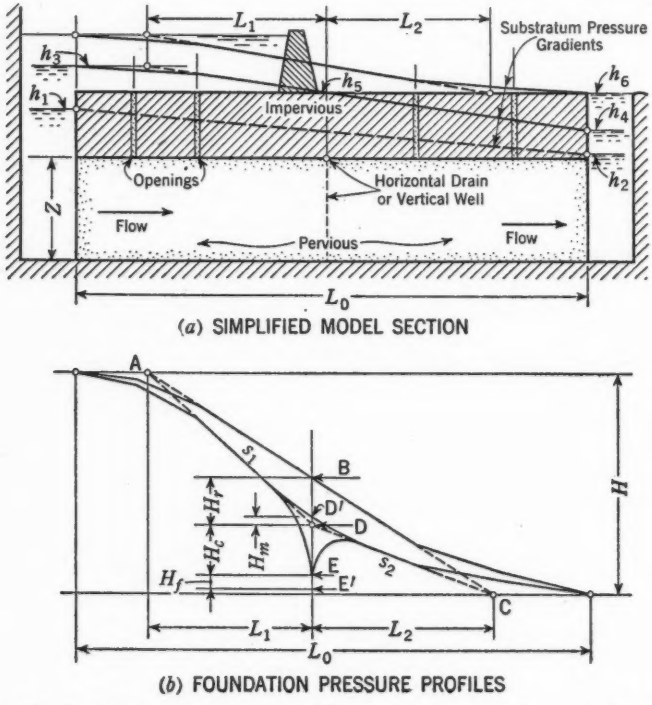


FIG. 28.—SIMPLIFIED MODEL SECTION (a), WITH FOUNDATION PRESSURE PROFILES FOR ASSUMED CONDITIONS

smooth curve which, theoretically, can be determined analytically.[32] Water flows through this system from the reservoir head, h_1, on the left to tailwater head, h_2, on the right. If both headwater and tailwater are below the top of the impervious cover, the effective length of the path of flow is L_0, and the pressure gradient is the straight line connecting h_1 and h_2 on the diagram. Assume that the headwater is raised to h_3, covering the impervious top stratum and causing inflow into the pervious lower stratum through the small openings. The pressure gradient is now broken, becoming steeper below each opening in proportion to the amount of seepage admitted. At this stage, the effective upstream length of the flow path could be determined by measuring the pressure gradient at the toe with several piezometers and dividing the head $(h_3 - h_1)$

[32] "The Effect of Blankets on Seepage through Pervious Foundations," by Preston T. Bennett, *Transactions*, ASCE, Vol. 111, 1946, p. 215.

by the observed gradient. Such a procedure would be identical with that shown by Fig. 27 for an actual piezometer installation.

The space below h_6 in the model represents underground storage in the prototype. As this storage fills up, seepage through the impervious cover begins at the outlets nearest the toe, and progresses landward. When the storage is exhausted, the effective land-side length of the model is established as L_2. Dimensions corresponding to L_1 and L_2 of the model have been established by piezometric observations at Fort Peck Dam, and have been used for fairly accurate predictions of relief well performance for all combinations of headwater and tailwater.[33] In a levee, where the "tailwater" elevation cannot be definitely established, and where filling of underground storage involves driving out compressible air from the foundation, the writer knows of no case where L_2 has been satisfactorily determined by piezometer observations. Field observations show a variation in L_2, just as it varies in the model, Fig. 28, before a steady state is established. In the absence of observational data, the distance L_2 may be estimated roughly from theoretical blanket formulas.[32]

After the equivalent paths of flow upstream and downstream from the proposed line of wells have been determined, the basic over-all flow patterns, without drainage relief, may be represented very simply by the foundation pressure profile, ABC of Fig. 28(*b*). Referring to the simplified model, and not the prototype, let it now be supposed that a narrow, fully penetrating slot is introduced into the pervious medium at point B, and that water is allowed to flow out of the slot through a standpipe with a variable discharge elevation represented in Fig. 28 by D. Assuming no head loss in the slot or standpipe, the former profile ABC is broken into two tangents AD and DC whose slopes, s_1 and s_2, are directly proportional to the flows approaching and leaving the pressure-relief slot. The flow from the drainage slot is, therefore:

$$Q_d = k_h Z (s_1 - s_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (53a)$$

or, in terms of the pressure reduction H_r,

$$Q_d = k_h Z H_r \left(\frac{L_1 + L_2}{L_1 L_2} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (53b)$$

These equations represent the performance of a hypothetical perfect drain, where no loss of head within the drain is involved. Any type of actual drain will involve a loss of head due to concentration of the flow into a drainage surface smaller than the sectional area of the pervious stratum. This loss of head is represented by H_c in Fig. 28(*b*). Suppose that the model flume, Fig. 28(*a*), is provided either with a vertical well or a horizontal drain as indicated by the dotted lines, and that the discharge elevation of the drain outlet is lowered to some elevation E′ (Fig. 28(*b*)) at which the drain discharge is equal to the open slot discharge given in Eqs. 53. The total friction loss in the system is H_f, and the head at the drain surface within the stratum is at elevation E.

The profile AD′C represents pressures along a straight line parallel to the axis of the model flume, and at the maximum distance from the drain. Such a

[33] "Pressure Relief Wells at Fort Peck Dam," by F. B. Slichter, Rept. on Conference on Control of Underseepage, U. S. Waterways Experiment Station, Vicksburg, Miss., April 1, 1945.

line would be midway between vertical wells, or at the bottom of the stratum if a horizontal drain were used. The profile AEC similarly represents horizontal flow gradients along a straight line passing through the drain. Beyond the region of influence of the wells (which from well formulas is not very extensive) the flow is evenly distributed and the pressure profiles AD'C and AEC coincide with ADC for most of their length. Near the drain, water moves laterally from the most remote path to the path through the drain, flattening the profile AD'C and making AEC steeper. Between the extremes AD'C and AEC there are an infinite number of intermediate horizontal flow gradients, but at any plane upstream from the drain the total horizontal flow is:

$$Q_1 = k_h Z s_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

and, consequently, the average horizontal gradient over the entire area of flow remains s_1. Similarly, the average horizontal gradient at any point downstream from the wells is s_2; and it follows that the average head in the vertical section through the drain is at D, exactly equal to the head in a fully penetrating open slot discharging an equal quantity.

This statement may seem so obvious that the foregoing discussion is unnecessary, but it expresses a basic idea which can be used to simplify the design of any deep pressure-relief system. The recommended procedure is briefly as follows:

(*a*) Establish the permissible subsurface pressure, D, using the criteria given by the authors. If any field comparisons of subsurface pressure with the actual occurrence of boils is available, such data should also be used.

(*b*) Estimate L_1 and L_2, or s_1 and s_2, using actual field observations if possible, by the general methods mentioned earlier in the discussion.

(*c*) Estimate the coefficient of transmissibility ($Z k_h$) as closely as possible, preferably using the Thiem tests if the cost of collection and disposal of the drain discharge is an important factor in the problem. At this stage, without going into final details of the type of drain to be used, the pressure relief H_r and the total drainage discharge Q_d can be determined, establishing the general features and cost of a collection and disposal system. It is quite possible that at this stage the economic feasibility of deep drainage can be determined.

(*d*) The location and grade of the collection system will establish the outlet elevations E' along the drainage system, and the last step of the design procedure becomes the choice of a drain type, size, and penetration which will discharge the required quantity Q_d within the available head ($H_c + H_f$).

If the pervious stratum is deep and has the normal degree of stratification, vertical wells appear to be the best type of drain. A horizontal drain may prove to be more economical if the pervious stratum is thin and homogeneous. In either case, well formulas similar to those given by the authors can be used.

From Morris Muskat's[34] basic solution of multiple-well problems, it has been shown that the head loss H_c can be expressed as:

$$H_c = (s_1 - s_2) F P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

[34] "The Flow of Homogeneous Fluids through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1937, Chapter IX.

in which

$$F = \frac{a}{2\pi} \log_e \frac{a}{2\pi r_w} \quad \ldots\ldots (56)$$

and P is a well penetration factor.

Since F is equal to the "extra length" factor used by the authors, it is evident that Eqs. 53 and 55 are equivalent to Eq. 13 when $s_2 = 0$. Thus, for the special case of no flow past the wells, the only difference involved is that the authors use the maximum pressure between wells as the permissible design value, whereas the writer suggests that the average pressure in the plane of the wells be used. For fully penetrating wells, the difference between maximum and average pressures, D'–D (Fig. 28(*b*)), is

$$H_m = a\,(s_1 - s_2)\,\frac{\log_e 2}{2\pi} = 0.11\,a\,(s_1 - s_2) \quad \ldots\ldots (57)$$

or

$$H_m = \frac{0.59\,H_c}{\log_e \dfrac{a}{2\pi r_w}} \quad \ldots\ldots (58)$$

The difference between average and maximum pressures is ordinarily so small that either may be used in determining allowable substratum pressures. The writer's preference for the average value is based solely on the fact that it is the "PI" of the tangent gradients s_1 and s_2 which fix the relationship between the drains and the remaining parts of the flow system.

For homogeneous strata, the factor P can be determined from the experimental data given by the authors. For stratified foundations, it should be determined by field tests in which the penetration is actually varied within the limits being considered. In the latter case, the factor would be:

$$P = \frac{\text{well drawdown for partial penetration, discharge } Q_w}{\text{well drawdown for full penetration, discharge } Q_w} \quad \ldots\ldots (59)$$

Fig. 28 suggests a method for defining the efficiency of a deep drainage system. If the system is designed so that the friction losses (H_f) are negligible, as recommended by the authors, the head E can be lowered to the tailwater elevation C. In this case, the efficiency of the system may be defined in terms of the ratio of head reduction to the head without drainage. It can be shown that the drain efficiency is:

$$\eta_d = \frac{H_r}{H_r + H_e} = \frac{L_2}{L_2 + F\,P} \quad \ldots\ldots (60a)$$

If L_2 is infinite, as assumed by the authors,

$$\eta_d = \frac{H_r}{H} = \frac{L_1}{L_1 + F\,P} \quad \ldots\ldots (60b)$$

Fig. 29 shows curves which may be used to determine readily the efficiency of a fully penetrating well system if the penetration factor P is known.

Well formulas may be used to evaluate a horizontal drain at the top of the pervious stratum by transposing vertical and horizontal dimensions in the plane

of the wells, and considering the horizontal drain to have half the discharge capacity of wells spaced a distance of 2 Z. If, as usually happens, the stratum is anisotropic, the equivalent well spacing used should be transformed to $2\,Z\,\sqrt{k_h/k_v}$, and the corresponding permeability $\sqrt{k_h\,k_v}$ should be used in computing the quantities of flow.

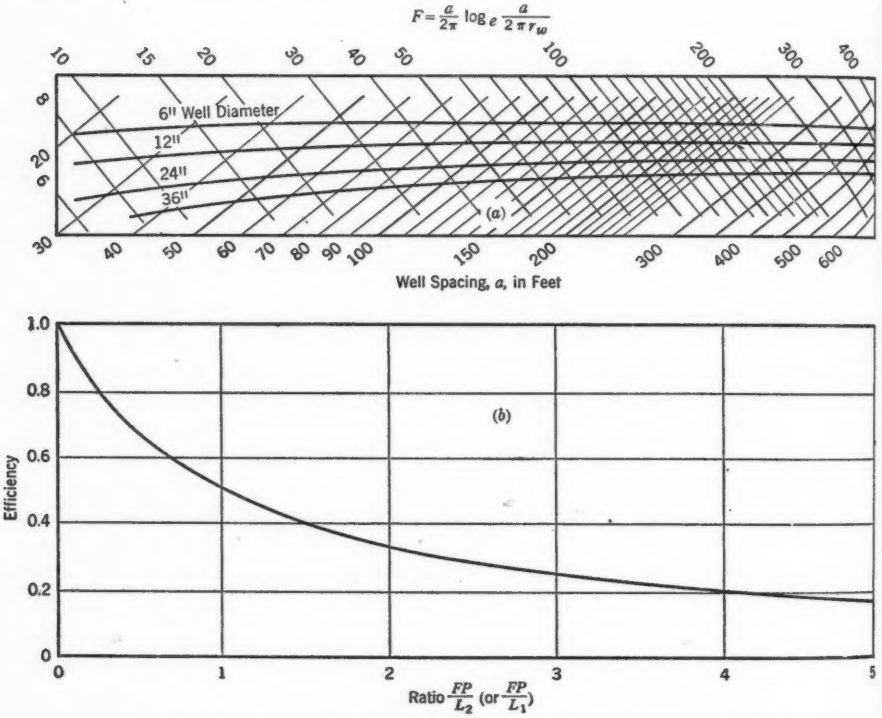


FIG. 29.—CURVES FOR DETERMINATION OF RELIEF WELL EFFICIENCY WHEN PENETRATION FACTOR P IS KNOWN

Eqs. 53 are directly applicable to drainage systems in which a continuous deep drainage zone might be used instead of vertical wells. It should be possible, by model studies or by analytic methods, to evaluate a factor for this type of drain which would be analagous to $F\,P$ in Eqs. 60, thus permitting a comparison in the efficiency of the two types.

It has been proposed to construct continuous deep drainage zones by excavation and backfill with highly pervious material, or by increasing the permeability of the sand by jetting. The writer has no knowledge on which to base any definite conclusion as to the practicability of materially increasing the permeability of the drainage zone. It would appear, however, that washing out any considerable part of the fines would be a very costly and difficult operation. If the jetting should succeed in breaking up the stratification without removing the fines, the maximum possible value of k_d would be the

mean permeability of the original materials, or

$$K_d \text{ (max)} \approx \sqrt{k_h k_v} < k_h \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

Even if a considerable volume of fines were removed, as would be necessary to make k_d greater than $\sqrt{k_h k_v}$, it is difficult to imagine a treatment that would make k_d equal to k_h. It appears that the drainage zone would still be anisotropic, with a horizontal permeability k_{dh} less than k_h, and a vertical permeability k_{dv} greater than k_v. If it is conceded that some flow landward from the drain always occurs in the prototype, and thereby gives some natural pressure relief, the possible decrease from k_h to k_{dh} within the drainage zone would form a slight obstruction to horizontal flow across the drain, and therefore would offset to some extent the pressure relief gained by increasing k_v to k_{dv}.

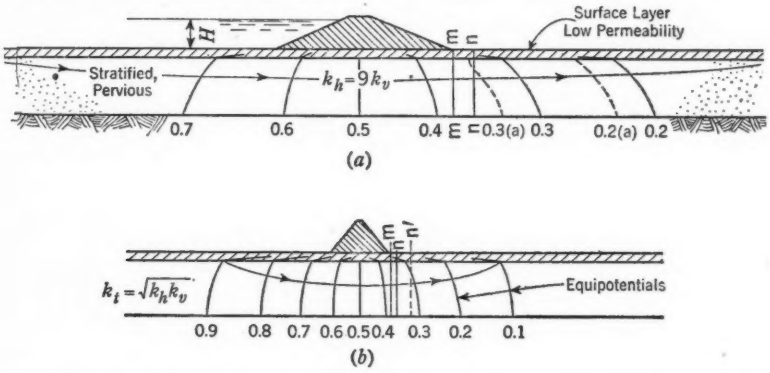


FIG. 30.—COMPARATIVE FLOW NETS FOR DAM OR LEVEE ON PERVIOUS FOUNDATION WITH $k_h = 9\,k_v$ FOR
(*a*) Natural Section
(*b*) Transformed Section

For the possible case $k_{dh} = k_{dv} = \sqrt{k_h k_v}$, the effectiveness of the drainage zone may be estimated by inspection of a flow net sketch. Fig. 30 represents a dam or levee on a pervious foundation in which $k_h = 9\,k_v$. The lines m-m and n-n represent limits of a potential drainage zone. If Fig. 30(*b*), all horizontal dimensions have been reduced in the ratio $\sqrt{k_v/k_h} = \frac{1}{3}$, and a flow net sketched in. This net has been replotted in Fig. 30(*a*) as shown by the solid equipotential lines, representing conditions without the drain. Foundation pressures in the region of the proposed drain appear to range from 35% to 38% of the total head. If the permeability of the drainage zone were altered to $\sqrt{k_h/k_v}$ the transformation in Fig. 30(*b*) would be accomplished by reducing all distances to the left of m-m and to the right of n-n, but leaving the interval between unchanged in scale, as shown by the line n′-n′ in the figure. Transposing the net back to the undistorted section gives the modified flow net indicated by dotted lines wherever it differs from the original. The pressure in the disturbed zone now appears to be from 30% to 36% of H, and the efficiency as previously defined would be about 15%. Actually, some additional pressure relief not indicated by the flow net would be gained by removal of the top stratum resistance, but the relief due to jetting alone is fairly well represented by the sketches.

CHARLES I. MANSUR,[35] JUN. ASCE.[35a]—The importance of the control of substratum pressures and seepage beneath hydraulic structures on pervious foundations, as outlined by the authors, cannot be overemphasized. Uncontrolled substratum pressure or seepage has frequently been a direct or contributing cause of the failure of structures that must withstand differential hydrostatic pressures. In addition to the use of pressure-relief wells on the land side of levees and dams as discussed in the paper, wells may also be used to provide pressure relief and drainage beneath other types of structures on pervious foundations. Some examples are stilling basins for outlet and spillway structures, locks, dry docks, weirs, floodgates, and drainage structures.

The primary function of a system of drainage wells is to provide pressure relief and drainage for water under hydrostatic pressure in a pervious substratum. If a drainage well is to operate efficiently as a means of relieving substratum pressures, it must be designed to: (*a*) Offer little resistance to the flow of water into and out of the well; (*b*) prevent infiltration of sand into the well after initial pumping; (*c*) resist the deteriorative action of the water and soil; and (*d*) withstand the earth pressure.

The material for the screen and riser pipe is a special item, which should be selected for each installation. Some of the factors that must be considered in the selection of a material are: (1) Corrosiveness of the soil and water; (2) expected operation of the wells—whether intermittent or continuous; (3) method of installation; (4) size and depth of well; (5) durability under water and alternate wetting and drying; (6) crushing strength; and (7) types of joints for joining the various sections. If practicable, all joints should be flush and smooth with the outside diameter of the pipe. A variety of materials which may be used as drainage wells are: Brass, wrought iron, aluminum, cast iron, galvanized iron, plastic, cement asbestos, bituminous fiber, concrete, clay tile, porous concrete, and wood. None of these materials should be used without first ascertaining whether they will withstand the corrosive action of the soil and water in which they will be placed. General experience has indicated that certain materials, such as brass, stainless steel, cast iron, and special metal alloys, clay tile, and wood, are to be preferred for drainage wells.

The wells, including screen, riser, and discharge pipe, should have an inside diameter which will permit maximum design flow without a head loss (friction and velocity) of more than $\frac{1}{4}$ ft to 1 ft of water, depending on the residual pressure permissible in the design. In no instance should the inside diameter be less than 2.5 in. From hydraulic considerations the screen section of wells up to 6 in. in diameter should have at least twenty-five holes or ten slots with an open area of at least 3 sq in. to 6 sq in. per lin ft of screen. For design purposes it is the writer's belief, based on test data, that the screen openings should be equal to, or less than, the 70% size of the foundation sand or filter gravel.

The joints in the screen and riser sections of the well should be designed and constructed, where practicable, in such a manner that they will support the weight of the pipe as it is lowered into the hole. When not practicable to construct the joints in this manner, the riser should be supported, while being

[35] Engr. (Soil Mechanics), U. S. Waterways Experiment Station, Corps of Engrs., Vicksburg, Miss.
[35a] Received March 31, 1947.

lowered into the casing, by some means which will insure that the joints will not open up. Where the screen and riser pipe consists of numerous loose joints (as, for example, with clay or concrete pipe) the top and bottom of the well should be joined with a brass or wrought-iron rod to eliminate or minimize the opening of any of the joints. All joints should have an overlap of at least $\frac{1}{2}$ in., and should be coated with a good quality asphaltic roofing cement or equivalent plastic.

Experience indicates that wooden pipe is one of the best materials for pressure-relief wells, providing the screen and riser pipe are submerged continuously. The screen of wooden wells may be perforated either with circular holes or slots. It is desirable that circular holes be seared smooth with a hot iron prior to installation. Wooden pipe may be slotted with a small circular saw. Various widths of slot may be obtained by using one or more circular saw blades.

Where metal pipes are used for relief wells, all parts of the screen, riser pipe, and discharge pipe should be of the same kind of metal to minimize corrosion by electrolysis. The results of field and laboratory tests indicate relatively little difference in the discharge efficiency of metallic well screens of the diameter that have not had an opportunity to corrode or clog. Laboratory tests, using a circular well-tank testing device, on six different new 3-in. commercial well screens, showed well-screen entrance losses of about 0.2 ft to 0.5 ft of water for a discharge of 5 gal per min per lin ft of well screen. A uniform, fine foundation sand was used in these tests.

Where filter gravels are to be placed around the screen part of the well, the perforations should be equal to, or less than, the 70% size of the filter gravel. Gravel filters around a perforated screen pipe should have a thickness of at least 4 in., measured radially from the outer circumference of the pipe. The filter material should be a washed gravel or a crushed stone composed of hard, tough, and durable particles. The gradation of the filter material will depend on the foundation sand being drained. The gravel should be of a size and gradation that will prevent "inwash" of any appreciable quantities of sand into the well, and also of a size and gradation which will be retained by the screen section of the well. The filter should also be considerably more pervious than the foundation sand. These filter requirements may be met by a filter which conforms to the following criteria:

$$\frac{\text{15\% size of filter}}{\text{85\% size of foundation}} < 4 \text{ to } 5 < \frac{\text{15\% size of filter}}{\text{15\% size of foundation}}$$

In addition to meeting these criteria, filters should be graded uniformly without any lack or excess of particles of any particular size. To minimize segregation during placement under water, the filter should be as uniform as possible and yet meet the aforementioned criteria.

There are many pervious foundations that are sufficiently well graded and which contain sufficient gravel to permit the installation of perforated well pipe without an artificial filter. Following installation, such wells should be "developed" by pumping and surging so as to build up a pervious filter around the well screen. The perforations in the screen for a well of this type should be of sufficient size to permit the inwash of sand within the immediate vicinity

of the well and yet to hold back the larger gravel particles, which thus build up a natural filter.

Porous concrete pipe may also be used as a filter around perforated pipe screens. If porous concrete pipe is used as a filter, it should be fitted closely around the inner pipe and should have a wall thickness of at least 1 in., and preferably greater. It should drain the foundation material freely without inwash of sand (this should be checked by laboratory tests prior to installation). The 70% size of aggregate used in the manufacture of the pipe should be greater than the perforations of the inner pipe.

The efficiency of various types of perforated pipes, with pervious gravel filters surrounding them, is illustrated by the results of tests in a well tank, using a fine, uniform foundation sand (see Table 3).

TABLE 3.—RELATIVE EFFICIENCY OF PERFORATED PIPE (FLOW, 5 GAL PER MIN PER LIN FT)

WELL PIPE		PERFORATIONS		FILTER	HEAD LOSS, IN FEET OF WATER	
Inside diameter, in inches	Material	Size and shape (in.)	No. per linear foot		Screen	Screen and filter
(1)	(2)	(3)	(4)	(5)	(6)	(7)
2½.....	Clay tile	$\frac{1}{16}$ ϕ	48	Porous concrete[a]	....	0.3 to 0.5
6.....	Wood	$\frac{3}{4}$ ϕ[b]	36	Gravel[c]	0.12	0.28
6.....	Wood	$\frac{1}{16}$ by 2½[b]	6	Gravel[c]	0.18	0.24

[a] 4-in. inside diameter pipe, 1-in. wall thickness. [b] Holes and slots seared with a hot iron. [c] Uniformly graded gravel from No. 20 sieve to ¼-in., filter 8-in. thick.

The casing for drainage wells may be installed by various methods; but the method used should be one which will not cause any excessive disturbance of the foundation sand outside the casing. Where filter materials are to be placed around the well screen, the bottom foot of the hole should be filled with the filter gravel prior to placing the well. The screen should then be properly centered in the casing and held securely in position while the filter is being placed and compacted, and while the casing is being withdrawn.

The filter material should be placed in small increments, so as to permit thorough "rodding" and not to interfere with the removal of the casing. To obtain a continuous filter, each increment of it should be compacted by rodding immediately after placing, and again after each increment of casing has been withdrawn. The casing should be withdrawn in an amount approximately equal to the depth of increment of filter placed. The alternate placing of filter and withdrawing of casing should be continued until the filter has been placed to the elevation of the top of the screen section and the casing has been withdrawn. The riser pipe should then be cut off at the elevation indicated for its termination, capped, and connected to the collector line or ditch in an approved manner. As pointed out by the authors the space around the riser pipe through the top stratum must be sealed carefully.

Promptly on completion of the installation of a well it should be pumped until a clear stream of water is obtained. Sometimes it is desirable to surge the well. When a well continues to produce sand it should be abandoned and a new well constructed near by. Abandoned wells should be plugged, and the riser pipe removed where possible.

Under the heading, "Design of Relief Well Systems," the authors state that a safety factor of 1.5 as applied to the critical gradient, $i_c = \frac{\gamma_s}{\gamma_w}$, may be satisfactory. The adequacy of this factor of safety depends, to a considerable extent, on the completeness of the field exploration of the foundation and top stratum. As illustrated by the authors, sand boils on the land side of dams and levees are the result of excessive hydrostatic pressures in a pervious substratum, which offers communication of pressure and seepage from the river or from borrow pits excavated to sand on the river side of the structure. If this subterranean pressure creates a hydraulic gradient sufficient to cause "flotation" of the soil in cracks, holes, or other thin or weak spots in the top stratum, sand boils will appear. The gradient required to cause such flotation may be considerably less than that theoretically required to cause flotation of the entire top stratum. As the foundation and top strata are usually nonhomogeneous in character, erosional seepage tends to concentrate in paths of least resistance to flow, causing sand boils to appear at localized spots instead of causing the entire foundation or top stratum to become "quick." Because of the foregoing reasons, effect of time on well-screen efficiency, and the other uncertainties of well design, a factor of safety of 2.0 may be warranted. If the flow from the wells is more than necessary to provide adequate pressure reduction, the wells can always be throttled down by valves, orifices, or other means; however, in some cases, conditions or time may not permit the installation of additional wells if the original design is not adequate.

At various times, there has been considerable discussion on the increase of total water on the land side of levees, occasioned by the installation of pressure-relief wells. The primary purpose of pressure-relief wells is to reduce the hydrostatic pressure on the land side, or the downstream side, of levees or dams. By so doing, the hydraulic gradient from the river or reservoir toward the land side is increased, of course, which therefore increases the landward flow a certain amount. The writer knows of no field observations regarding the amount of such increase of land-side seepage; but the following example, based on the theory of critical gradient as presented by the authors, and on a study of seepage models by the writer, indicates the approximate increase of landward seepage that would be caused by drainage wells. Begin with four assumptions: A water height, H, of 30 ft; a top stratum depth of 8 ft; a critical gradient, i, of 0.9; and a saturated foundation.

The flow through a pervious stratum beneath a levee is directly proportional to the head drop from the river side to the land side. The maximum hydrostatic pressure that can exist land side of the levee would be $h_2 = 8 \times 0.9 = 7.2$ ft. (Any substratum pressure greater than this would raise the land-side blanket.) Therefore, the net hydrostatic head causing landward flow would be $\Delta H = 30 - 7.2 = 22.8$ ft. The flow resulting from this head drop

will emerge somewhere landward of the levee as general seepage, or as flow from sand boils. If a sufficient number of drainage wells were installed to lower the residual pressure on the land side to, say, $h'_2 = 1.0$ ft, the landward flow would be that corresponding to a pressure drop of $\Delta H' = 30 - 1 = 29$ ft. In other words, the drainage wells would increase the landward flow by $\left(\frac{29.0 - 22.8}{22.8} \times 100\right) = 27\%$. The total flow that must be cared for by the surface drainage system land side of levees, during flood time, is composed of surface runoff occasioned by rainfall, general seepage, and the additional flow that would result from the installation of some drainage system. On the basis of the foregoing computations, it may be seen that the additional flow resulting from drainage wells would be relatively small in most instances, considering the total flow that must normally be cared for by the surface drainage system.

The authors have very thoroughly covered the theoretical design of relief wells. Proper design of well spacing, diameter, and penetration is of primary importance in a well system. Of equal importance in the design of a relief well system is the design of the individual wells and their installation because, unless the wells function properly, the best of theoretical designs will not accomplish their purpose. Therefore, some additional discussion of this aspect of well design and installation was considered warranted.

S. J. JOHNSON,[36] JUN. ASCE.[36a]—In bridging the gap between the mathematician and the engineer in a practical manner, the authors have rendered a service to the engineering profession. The design of relief well systems has been greatly simplified by the design curves in this paper, without which the solution of the fundamental design equations becomes a time-consuming operation. Relief well systems are assuming such importance that the presentation of this paper is indeed timely.

The mathematical development presented by the authors is for purely artesian flow. A formula similar to Eq. 7*a* has been developed by Phillipp Forchheimer[28] for gravity flow in a homogeneous soil, based upon J. Dupuit's assumption that the gradient on any vertical is constant and equal to the slope of the seepage surface in the flow direction. The depth of saturation h at any point is given by:

$$h^2_s - h^2 = \frac{Q_g}{2\pi k} \log_e \left[\frac{\cosh j\,(y + s) - \cos j\,x}{\cosh j\,(y - s) - \cos j\,x}\right] \ldots\ldots\ldots (62)$$

in which h_s is the depth of saturation at the line-drive source;[36b] Q_g, the gravity flow discharge out of each well, is:

$$Q_g = \frac{2\pi k\,(h^2_s - h^2_w)}{2 \log_e \frac{e^{js}}{j\,r_w}} \ldots\ldots\ldots (63)$$

---

[36] Engr., U. S. Waterways Experiment Station, Corps of Engrs., Vicksburg, Miss.

[36a] Received March 31, 1947.

[28] "Hydraulik," by Phillipp Forchheimer, B. G. Teubner, Leipzig and Berlin, 1930.

[36b] Correction: In Eq. 7*a*, change *d* to *s* in two places. For other corrections, read p. 235, February, 1947, *Proceedings*.

The writer has had occasion to use the gravity equations in studies for a dam that rests on a pervious foundation possessing a natural impervious top layer. Over the downstream part of the foundation, this top layer had been removed to permit foundation drainage, which was hindered by foundation stratifications by-passing part of the underseepage. This condition created undesirable uplift pressures at and beyond the downstream toe of the dam. To study the effect of pressure-relief wells, the gravity discharge, Q_g, was equated to the artesian seepage, Q_a, that existed between the upstream end of the natural blanket and the upstream end of the stripped dam foundation, thus:

$$Q_g = \frac{2\pi k (h^2_s - h^2_w)}{2 \log_e \frac{e^{is}}{j r_w}} = Q_a = \frac{k a d (\Delta H + h_w - h_s)}{L} \quad \ldots\ldots (64)$$

—to obtain h_s which is the head at the upstream end of the stripped foundation. In Eq. 64 ΔH is the difference in elevation between reservoir pool and tailwater of the well discharge (head losses in the well considered to be negligible), and L is the distance between the upstream end of the natural blanket and the upstream end of the stripped foundation. With h_s thus determined, the head at any desired location may be determined from Eq. 62. For conditions studied, the head h_s is slightly less, and the heads midway between wells are slightly more (as computed by the foregoing method), than those given for similar points by Eq. 7*a*.

An important item, as the authors have indicated, is the determination of the effective distance from the source to the line of wells. Information on this factor can be determined in advance, when the wells are to be installed behind existing dams or levees, by placing piezometers at various distances from the dam or levee on the water side. Another method is to evaluate this distance by mathematical means[32] after first determining the permeability and extent of the permeable stratum and top stratum. This latter method must be used when piezometer observations cannot be made. The effective distance from the source can also be determined for existing installations if the quantity of flow is known, together with the permeability and the dimensions of the foundation soils. These three methods were used by the writer in analyzing data obtained from a drainage well installation at a flood control dam. The methods gave results which were all the same degree of magnitude, although there were some differences, which probably can be explained by the variations in field conditions which are generally present.

The behavior of the toe drainage provisions at one dam was studied by the U. S. Waterways Experiment Station (Vicksburg, Miss.) in January, 1945. The effect of closing the wells on the piezometric pressures in the line of wells was very pronounced, since the head above ground surface increased from about 1.3 ft with the wells in operation to 6.1 ft with the wells closed. Inasmuch as the natural blanket, or top stratum, was only 8 ft thick at the section where these readings were taken, the necessity of maintaining the wells in good operating condition is evident. Tests such as described herein obviously should be made only under closely controlled conditions.

An important point, not considered by the authors, is the reduction of well discharge rates after installation. The Waterways Experiment Station has studied the reduction in well discharge of the Sardis experimental well installation. A study of forty-six wells completed in May, 1942, showed that, assuming the initial output as 100%, after two years the average output of all wells had dropped to 35%. However, surging of the wells at that time increased the average well output to 56% of the original output. The comparisons were made at the same net head. These reductions may be caused by one or both of the following: (1) Silting up of the bottom of the reservoir; and (2) clogging of the well screen or filter. Inasmuch as the reservoir is fed by a muddy river some silting probably occurs, although the total quantity of seepage flow has not decreased materially according to other data available. The general improvement of the wells after surging indicates that the major cause of discharge reduction is screen clogging. Such clogging can be caused by imperfect filter action or by chemical deposition.

The experimental wells were pulled during December, 1946, and the screens revealed corrosion and deposition, especially those with fine screen or mesh openings and those that were composed of iron pipes with brass screens or mesh. The ground water at this site has a low pH-value and a sufficient amount of dissolved CO_2 to render it corrosive. The ratio of the 85% size of the sand or gravel immediately adjacent to the screen to the size of screen opening varied from 1.0 to 1.4 for the nonmetallic pipe; whereas the ratio of the 15% size of the filter gravel to the 85% size of the foundation sand varied from 2.3 to 3.0. Some inwash of sand occurred during surging after initial installation. In view of the foregoing satisfactory ratios and the variations of the well discharges, it is evident that there was either a progressive clogging of the well screen, or filters, or both. The surging in June, 1944, materially increased the flow, and thus it probably partly cleared the screen clogging—but not entirely. In general, it is desirable to keep the entrance velocity through the screen openings as low as possible, since the change in pressure head outside the screen to velocity head in the screen releases CO_2 and other gases, thereby increasing corrosion and incrustation.

The useful life of water-supply wells was the subject of six papers presented at the 1946 Annual Conference of the American Water Works Association, and which have been reported in abbreviated form.[37] These papers presented factual data showing that screen clogging and incrustation are common and that a reduction in well output with time is to be expected to some degree. "The average life of screened wells under all conditions seems to be from 25 to 30 years," according to E. W. Bennison, M. ASCE, and "Sufficient length of experience is not available to forecast the life of gravel-packed wells but it is believed one-half of the value is gone in 15 years," according to A. D. Henderson. Although the conditions surrounding the use of water-supply wells may be different from those surrounding the use of relief wells, the data mentioned in this discussion are at least indicative.

The experiences cited herein emphasize the necessity for designing pressure-relief wells oversize to provide for possible reduction in capacity, and to pro-

[37] *Journal*, A. W. W. A., January, 1947.

vide for easy replacement of wells at the expiration of their useful life, regardless of whether that be five or fifty years. From the data and experience to date, it appears necessary to consider carefully the materials of which the wells are to be made, and the ground water and soil conditions at the site.

In the experimental well system at Sardis Dam (Mississippi) it was found (as the authors have stated) that wooden wells performed satisfactorily. In this connection, it is interesting to record that slotted wooden wells have been used for water supply purposes for many years in Holland, and with satisfactory results. The wells used in that country are made of hard wood. The wooden wells may be obtained with thin slots parallel to the grain, which are about $\frac{1}{16}$ in. wide, 3 in. long, and spaced at intervals of approximately $\frac{1}{2}$ in. around the outside of the pipe. They may also be obtained with slots perpendicular to the grain but inclined upward, which would tend to minimize the possibility of sand being carried into the well.

One item, which the writer believes to be especially important, is the determination of the discharge elevation of the wells. In general, it is highly desirable that this elevation be as low as possible. Considerable care should be given to this decision, as it may mean the difference between a system which is entirely satisfactory and one which is not.

Gravel filters artificially placed around relief wells are necessary if an adequate natural filter is not developed by pumping and surging of the well. The relative merits of these two types of filter are somewhat controversial; however, the use of one or the other is considered essential.

The writers are to be congratulated for bringing the design of relief well systems to the attention of engineers in a timely and well-prepared paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

A RATIONAL EXPLANATION OF COLUMN BEHAVIOR

Discussion

BY R. ROBINSON ROWE, AND GEORGE WINTER

R. ROBINSON ROWE,[6] M. ASCE.[6a]—The use of the more precise expression for the elastic curve of a uniform column, instead of the usual approximation, should be accompanied by a few words of its history and analogous applications in engineering. The particular curve is known as the "arch of Yvon-Villarceau" from the name of the mathematician who according to Rankine first investigated its properties by the use of elliptic functions. It is the curve whose curvature is proportional to distance from a directrix, or simply,

$$\rho\, y = \text{constant} \qquad (53)$$

James Bernoulli discovered this shape in the seventeenth century in his study of an indefinitely broad sheet supporting a liquid mass. This is the "hydrostatic cord" assumed by sheet metal flumes; and the constant is the unit tension divided by the unit weight of the liquid. Formulas equivalent to Eq. 19*d* for seven cases have been developed by J. B. Macphail,[7] M. ASCE.

Yvon-Villarceau found the same shape inverted for his arch, designed to support a fluid-like earth without moment. This is also called the "hydrostatic arch." The constant, of course, is the unit compression, or thrust, divided by the unit weight of earth. His formulas in elliptic functions and many special properties of the curve are quoted by Rankine.[8]

Rankine noted the analogy between the hydrostatic arch, the hydrostatic cord, and the elastic curve for a spring or column under the action of a pair of equal and opposite end forces.[9] Eq. 3 by Rankine is equivalent to the author's Eq. 3 and Rankine's Eq. 7, to the author's Eq. 27*b*. In addition to the author's six cases (Table 2), Rankine showed several representative forms of undulating

NOTE.—This paper by Frederick L. Ryder was published in March, 1947, *Proceedings*.

[6] Senior Bridge Engr., State Div. of Highways, Sacramento, Calif.

[6a] Received April 2, 1947.

[7] "*The Engineering Journal*, December, 1934, p. 528.

[8] "Applied Mechanics," by W. J. M. Rankine, 4th Ed., pp. 190-195.

[9] *Ibid.*, p. 353.

distortion, which can actually develop in free-ended columns with intermediate lateral support. He did not repeat the elliptic expressions for the curve; but he referred to those for the hydrostatic arch, and noted that the constant was $\frac{EI}{P}$ and the directrix, the neutral posltion of the column.

GEORGE WINTER,[10] M. ASCE.[10a]—An elegant and interesting analytical treatment of some general features in the theory of elastic stability is presented in this paper. It will be of value to men interested in this field, primarily as a systematic presentation of known facts. It is unusual, in a paper covering such a vast and established field, not to find a single reference to previous work of other investigators. Indeed, the only references given relate to texts in the field of pure mathematics, concerning the analytical tools used in the derivations.

The origin of the analysis of columns beyond the Euler critical load goes back to Bernoulli and Euler, in the first half of the eighteenth century. Complete treatments of this problem of the so-called "Elastica" are found in such standard texts as A. E. H. Love's "Mathematical Theory of Elasticity" and R. V. Southwell's "Introduction to Theory of Elasticity." In the latter work Professor Southwell presented material which is not only identical in substance and mathematical method with the author's treatment,[11] but he includes a chart [12] that gives all the configuration shown by Mr. Ryder in Table 2. Furthermore, the information in Table 1 is given by Professor Southwell with some data added.[13] To be sure, there are differences of emphasis and detail, but none in substance and method.

In his discussion of practical columns and beam columns by the small deflection theory (differential Eq. 33), the author offers no information that cannot be found in generally available publications. In particular, Eq. 34 merely represents a superposition of customary solutions of beam columns as presented by S. Timoshenko,[14] D. H. Young,[15] Jun. ASCE, and others. The derivation of the Euler load from a beam-column analysis, Eq. 36, can be found in any standard textbook.

Numerical methods of integration applicable to columns and beam columns have been developed by N. M. Newmark, M. ASCE (among others). Professor Newmark indicates[16] how his method can be extended into the large deflection range; although, in view of the purely academic interest of this problem, he omits its elaboration. These numerical procedures are identical in substance, although not necessarily in detail, with those used by the author.

It is not implied in this review of the literature that the author's material is not new in some of its details and elaborations and that it was not independ-

[10] Associate Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

[10a] Received April 2, 1947.

[11] "An Introduction to the Theory of Elasticity," by R. V. Southwell, Oxford Univ. Press, 2d Ed., 1941, pp. 429-436.

[12] *Ibid.*, p. 430, Fig. 112.

[13] *Ibid.*, pp. 434-435, Tables 1 and 2.

[14] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, Chapter I.

[15] *Transactions*, ASCE, Vol. 101, 1936, p. 422.

[16] *Ibid.*, Vol. 108, 1943, p. 1187.

ent work that led him to his findings. However, the fundamental content of this paper and its methods, as well as its conclusions, have been part and parcel of established knowledge for a considerable time.

With regard to its practical implications, it should be noted that the major part of this paper deals with phenomena beyond the Euler critical load. In the first sentence of section 5, however, the author himself notes that "Columns loaded above the critical point, of course, would never be used as structural members." This substantial part of the paper, therefore, is of no practical help to the designing engineer. As far as columns loaded below the critical point are concerned, Fig. 6 is misleading; because practical columns do not behave in the way assumed for this figure, as is firmly established by numerous experimental and analytical investigations. Initial imperfections as well as nonelastic behavior for shorter columns make this graph inapplicable. On the other hand, the methods of dealing with practical columns as proposed by the author in section 4 (beginning at Eq. 32) have long been used in practical design. The fact that in this approach "there is no critical point for an actual column" is again part and parcel of these methods.

It is generally realized that the field of column analysis and design is in an unsatisfactory state. This situation, however, is not due to lack of analytical knowledge on the behavior of isolated columns in the elastic range. Indeed, the details of this topic have been worked out in the most exhaustive manner; and the author's paper, dealing exclusively with such isolated, elastic columns, adds no new evidence of value in the practical column range.

The real and substantial problems in column design, on the solution of which further progress depends, are (*a*) the behavior of isolated columns beyond the elastic range, and (*b*) the analysis of columns as component parts of structures (trusses, frames, bents) both in and beyond the elastic range. It is hoped that future investigators will turn their attention to these topics, which are of fundamental importance for the development of more accurate and rational structural column design.